Introduction: At the Intersection of Truth and Falsity

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'Now we will take another line of reasoning. When you follow two separate chains of thought, Watson, you will find some point of intersection which should approximate to the truth.'—Sherlock Holmes, in 'The Disappearance of Lady Frances Carfax'.

1. TOWARDS THE INTERSECTION

Suppose that we have (at least) two categories $\mathcal{X}$ and $\mathcal{Y}$ for any meaningful, declarative sentence $\mathcal{A}$ of our language.[1] Pending further information about $\mathcal{X}$ and $\mathcal{Y}$, there seem to be four options for an arbitrary sentence $\mathcal{A}$:

- $\mathcal{A}$ is only in $\mathcal{X}$
- $\mathcal{A}$ is only in $\mathcal{Y}$
- $\mathcal{A}$ is in both $\mathcal{X}$ and $\mathcal{Y}$
- $\mathcal{A}$ is in neither $\mathcal{X}$ nor $\mathcal{Y}$

Whether each such 'option' is logically possible depends not only on our logic (about which more below) but on the details of $\mathcal{X}$ and $\mathcal{Y}$.

Suppose that $\mathcal{X}$ comprises all (and only) sentences composed of exactly six words, and $\mathcal{Y}$ those with exactly nineteen words. In that case, only the third option is ruled out: $\mathcal{X}$ and $\mathcal{Y}$ are *exclusive*—their intersection $\mathcal{X} \cap \mathcal{Y}$ is empty—since no $\mathcal{A}$ can be composed of exactly six words and also be composed of exactly nineteen words.[2] Despite being exclusive, $\mathcal{X}$ and $\mathcal{Y}$ are *not exhaustive*—their union $\mathcal{X} \cup \mathcal{Y}$ does not exhaust all sentences—since some $\mathcal{A}$ may fall into neither $\mathcal{X}$ nor $\mathcal{Y}$. (Just consider 'Max sat on Agnes'.)

Consider another example. Let $\mathcal{X}$ comprise all sentences of your favourite novel and $\mathcal{Y}$ your all-time favourite sentences. In that case, exclusion is not ruled out; the intersection of $\mathcal{X}$ and $\mathcal{Y}$ may well be non-empty. (Suppose that your favourite sentence is the first sentence of your favourite novel.) Presumably, $\mathcal{X}$ and $\mathcal{Y}$ are not

[1] Henceforth, 'sentence' is used for meaningful, declarative sentences.

[2] Actually, even this is a bit contentious, since there are inconsistent (but non-trivial) arithmetics in which 19 and 6 'collapse'. (See [29].)

exhaustive, since (presumably) there are sentences that are neither your favourite nor in your favourite novel.

2. AT THE INTERSECTION

Now for the interesting question. Assuming that *truth* and *falsity* are categories of sentences, we can let $\mathcal{X}$ be the former and $\mathcal{Y}$ the latter. Let us assume, following standard practice, that one constraint on *falsity* is that, by definition, falsity is truth of negation, that is, that $\mathcal{A}$ is false if and only if its negation $\neg\mathcal{A}$ is true. The question, then, is this: Are $\mathcal{X}$ and $\mathcal{Y}$ both exclusive and exhaustive categories?

For present purposes, the question of exclusion is central.[3] Are truth and falsity exclusive? The question is intimately connected with others:

» Is there any a priori (or empirical) reason to think that truth and falsity are exclusive?

» If truth and falsity are exclusive, how is the non-exclusivity to be formulated? If truth and falsity are not exclusive, how is that to be formulated?

» How would we decide whether truth and falsity are (non-)exclusive? Can there be any non-question-begging debate?

» Is there any a priori (or empirical) reason to think that truth and falsity are not exclusive?

» Even if truth and falsity are not exclusive, is it rational to believe anything that lies in the intersection of truth and falsity?

I will not (here) address all of those questions; they are discussed in depth, in one form or another, in the following chapters.[4] Here my aim is (briefly) to cover a few topics that serve as background to the rest of the book. I give indications for further reading along the way.[5]

3. 'THE' LAW OF NON-CONTRADICTION

The classic source of much thought about contradiction comes from Aristotle's Book Γ of the *Metaphysics*. To this day, many of Aristotle's views have been widely rejected; the conspicuous exception, despite the work of Dancy [21] and Łukasiewicz [28], are his views on contradiction. That no contradiction is

[3] The two questions, as RESTALL, BRADY, and VARZI emphasize, are closely related, but I will concentrate on the question of exclusion in this introductory essay. MCGEE's essay also brings out the very tight connection between the questions of exclusion and exhaustion.

[4] In fact, the questions roughly correspond to the five parts of the volume.

[5] In giving further reading, I also highlight the chapters in this volume by using UPPERCASE for names of contributors.

true remains an entrenched 'unassailable dogma' of Western thought—or so one would think.[6]

In recent years, due in no small measure to progress in paraconsistent logic (more on which in ss. 4 and 7), the 'unassailable dogma' has been assailed. As Priest's detailed discussion shows [32], neither Aristotle's arguments for (non-)contradiction nor modifications of those arguments [3, 41, 45] have produced strong arguments for the thesis that no contradiction could be true—that the intersection of truth and falsity is necessarily empty. Moreover, there seem to be reasons for thinking that at least some contradictions are true (see s. 5). At the very least, the issue is open for debate—the main motivation behind this volume.

But what exactly *is* the so-called law of (non-)contradiction? Unfortunately, 'the' so-called law is not one but many—and perhaps not appropriately called a 'law'. Aristotle distinguished a number of principles about (non-)contradiction, and the correct exegesis of his views remains an issue among historians. For present purposes, I will simply list a few principles, and then briefly fix terminology concerning 'contradiction'.[7]

» Simple (Non-)Contradiction: No contradiction is true
» Ontological (Non-)Contradiction: No 'being' can instantiate contradictory properties
» Rationality (Non-)Contradiction: It is irrational (knowingly) to accept a contradiction

The principles, so formulated, are hardly precise, but they indicate different (not to say logically independent) versions of 'the' target principle. For present purposes, I will focus almost entirely on Simple (Non-)Contradiction, though some of what follows will also indirectly touch on the other principles.[8]

What needs to be clarified is the sense of 'contradiction' at play (at least in this introductory chapter). I will discuss two uses of the term, the *explosive* and the *formal* usage.[9]

Explosive Usage

Some philosophers use the term 'contradiction' to mean an *explosive sentence*, a sentence such that its truth entails triviality—entails that all sentences are true.

[6] Despite showing the holes in Aristotle's various arguments on (non-)contradiction, Łukasiewicz [28] concludes that Aristotle was right to preach (as it were) the 'unassailable dogma', as Łukasiewicz called it.

[7] Chapters by Brady, Restall, and Varzi are particularly relevant to the issue of formulating 'the' relevant 'law'.

[8] The chapters by Kroon, Cogburn, and Tennant are particularly relevant to all three principles, as is Brown's.

[9] Grim's chapters is particularly useful for gaining a sense of the divergent uses of 'contradiction', as is that by Weir.

A familiar example of such a sentence is 'Every sentence is true.' That sentence is apparently explosive, since if 'every sentence is true' is true, then every sentence is true, in which case triviality abounds.

Could a contradiction in the explosive sense be true? The question is tricky, as tricky as the modality 'could'. Suppose that by 'could' we mean *logically possible*. Then the question is: Is it logically possible that a contradiction (in the explosive sense) be true?

The answer, of course, depends on the given logic. Does classical logic afford the logical possibility of true contradictions (in the explosive sense)? Interestingly, there is a sense in which classical logic—or, at least, an intuitive account of classical consequence—does afford the logical possibility of true (explosive) contradictions.[10] Intuitively, an argument is classically valid if and only if there is no 'world' in which the premisses are true but the conclusion is untrue. Such worlds, on the classical account, are complete and consistent, in the sense that for any world $\mathcal{W}$ and any sentence $\mathcal{A}$, either $\mathcal{A}$ or its negation $\neg\mathcal{A}$ is true at w, but not both $\mathcal{A}$ and $\neg\mathcal{A}$ are true at w. What the classical approach demands, of course, is that if *both* $\mathcal{A}$ and $\neg\mathcal{A}$ are true at some world w, then so too is $\mathcal{B}$, for *any* $\mathcal{B}$. But, then, there is nothing in the classical account, at least intuitively understood, that precludes recognizing an exceptional 'trivial world', the world in which *every* sentence is true. In that respect, even classical consequence affords the logical possibility of true (explosive) contradictions: it is just the 'logical possibility' in which every sentence is true—the 'logical possibility' in which explosion happens!

Be that as it may, classical consequence is usually understood in terms of 'classical interpretations'. A classical interpretation is—or is usually modelled by—a function ν from sentences into $\{1, 0\}$ (intuitively, The True and The False) such that $\nu(\neg\mathcal{A}) = 1$ exactly if $\nu(\mathcal{A}) = 0$. But, then, there is no classical interpretation on which a contradiction (in the explosive sense) is true.

The upshot is that if classical logic dictates the space of logical possibility, there is at best only a remote and trivial sense in which contradictions, in the *explosive* sense, could be true. But there is another sense of 'contradiction', to which I now turn—and classical logic, of course, is only one among many logical theories.

Formal Usage

The explosive usage is not the only prevalent usage of 'contradiction', and for present purposes, it is not the target usage. The *formal* usage of 'contradiction' has it that contradictions are sentences *of the form* $\mathcal{A} \wedge \neg\mathcal{A}$, where $\wedge$ is conjunction and, as above, $\neg$ is negation. In other words, a contradiction, on the formal usage, is a conjunction of a sentence and its negation.

Tradition distinguishes between (among others) sub-contraries and contradictories. $\mathcal{A}$ and $\mathcal{B}$ are *contraries* if they cannot be true. $\mathcal{A}$ and $\mathcal{B}$ are *subcontraries* if

[10] Here, I assume single-conclusion classical semantics. As Greg Restall pointed out (in conversation), the issue is slightly more complicated in a so-called multiple-conclusion framework.

they cannot both be false. $\mathcal{A}$ and $\mathcal{B}$ are *contradictories* if they are both contraries and sub-contraries.

For present purposes, all that is required of a contradiction, at least on the *formal usage* (as here specified), is that it be of the form $\mathcal{A} \wedge \neg\mathcal{A}$. In particular, there is no further requirement that $\mathcal{A} \vee \neg\mathcal{A}$ be logically true, or that $\neg(\mathcal{A} \vee \neg\mathcal{A})$ be logically true.[11]

The target sense of 'contradiction' is the formal one.[12] Could such a contradiction be true? At this stage, the question of logic becomes pressing. If we let classical logic dictate the constraints of 'could' (in whatever sense might interest us), then we have already been through the question at hand. After all, if classical logic dictates the constraints of (say) logical possibility, then any *formal contradiction* is an explosive contradiction, as the famous 'independent argument' shows. (See s. 4 for further discussion.) But, as above, classical logic is just one among many different theories of consequence (validity). In addition to classical logic, and particularly relevant to the present volume, is so-called paraconsistent logic, to which I turn.[13]

4. WEAK AND STRONG PARACONSISTENCY

The question at the intersection of truth and falsity is whether it (the intersection) could be non-empty but non-trivial—whether *some but not all* contradictions could be true. Classical logic, and intuitionistic logic, for that matter, give a swift answer: No.[14] In each such logic, the so-called 'independent argument' goes through:[15]

(1) Assume that $\mathcal{A} \wedge \neg\mathcal{A}$ is true

(2) By (1) and Simplification, $\mathcal{A}$ is true

[11] Of course, one might argue—and some [40] have—that an operator φ is *negation* (or a negation) only if $\mathcal{A} \vee \varphi\mathcal{A}$ and $\varphi(\mathcal{A} \wedge \varphi\mathcal{A})$ are logically true. If that is right, then $\mathcal{A} \wedge \neg\mathcal{A}$ is a contradiction only if $\mathcal{A}$ and $\neg\mathcal{A}$ are sub-contraries and $\neg(\mathcal{A} \wedge \neg\mathcal{A})$ is logically true—since otherwise $\neg$ wouldn't be a negation. (Recall that on the formal usage, a contradiction is of the form $\mathcal{A} \wedge \neg\mathcal{A}$, where $\neg\mathcal{A}$ is the negation of $\mathcal{A}$.) But, again, I will leave this issue aside, not because it is not important but, rather, because a full discussion would be too full for present purposes. Useful discussion of negation is in Brady's paper, as well as Sainsbury's, and also in the volumes [23, 47] and Routley and Routley [44].

[12] Henceforth, I use 'contradiction' along the formal usage, unless otherwise specified.

[13] I will say nothing here about 'revisions of logic' or the like, due only to space considerations. My own view is along Quine-the-good lines, according to which *any* 'logical principle' may be revised in the face of appropriate 'evidence'. (Quine-the-bad, of course, imposed exceptions—notably, the 'unassailable dogma' of which Aristotle and Łukasiewicz spoke.) Resnik's chapter, in addition to those by Bueno and Colyvan and Brown, discuss these issues along various lines. The two letters by Lewis are also relevant.

[14] Priest [38] and Beall and van Fraassen [18] provide introductory presentations of intuitionistic logic, in addition to the sample paraconsistent framework discussed in s. 7. Priest's text also discusses more mainstream approaches to so-called relevant (-ance) logic.

[15] The 'proof' is often ascribed to C. I. Lewis, who rediscovered it for contemporary readers, but Medieval logicians were apparently aware of the proof (like so many other 'recent discoveries'). I am grateful to Graham Priest on the historical point.

(3) By (2) and Addition, $\mathcal{A} \vee \mathcal{B}$ is true

(4) By (1) and Simplification, $\neg\mathcal{A}$ is true

(5) But, then, by (3), (4), and Disjunctive Syllogism, $\mathcal{B}$ is true

The upshot is that any contradiction is explosive if each of the foregoing steps is valid.

Paraconsistent logics, by definition, are not explosive. A consequence relation $\vdash$, however defined, is said to be *explosive* if $\mathcal{A}, \neg\mathcal{A} \vdash \mathcal{B}$ holds for arbitrary $\mathcal{A}$ and $\mathcal{B}$. A consequence relation is said to be *paraconsistent* if and only if it is not explosive.[16]

A sample paraconsistent logic is presented in s. 7. That sample is one among various approaches to paraconsistent logic, and by no means decidedly 'the right one'. One approach, for example, due to Da Costa [19, 20], is to let negation fail to be truth-functional. Without truth-functionality, there is no a priori reason that $\mathcal{A}$ and $\neg\mathcal{A}$ could not both be true. Other approaches filter out explosion while retaining as many familiar features of the logical connectives as possible. And there are yet other approaches.[17]

Paraconsistent logic, regardless of the details, affords the 'possibility' of inconsistent but non-trivial theories—theories according to which both $\mathcal{A}$ and $\neg\mathcal{A}$ are true (for some $\mathcal{A}$) but not every sentence is true. Such logics, in other words, open up the 'possibility' in which *some but not all* contradictions 'could' be true.

The matter (again, regardless of the formal details) is delicate. Paraconsistentists, those who construct or use or rely on some paraconsistent logic, usually divide into (at least) three classes:

» Weak Paraconsistentist: a paraconsistentist who rejects that there are 'real possibilities' in which a contradiction is true; paraconsistent models are merely mathematical tools that prove to be useful but, in the end, not representative of real possibility

» Strong Paraconsistentist: a paraconsistentist who accepts that there are 'real possibilities' in which contradictions are true, and more than one such 'real possibility' (and, so, not only the trivial one); however, no contradiction is in fact true

» Dialetheic Paraconsistentist: a paraconsistentist who accepts that there are true contradictions—and, so, that there could be (since our world is a 'real possibility' in which there are some)[18]

Most contemporary paraconsistentists, including so-called relevantists [1, 2, 43], fall into the first class. The minority position, but the position of most relevance

[16] That account of paraconsistent consequence is not ideal, but it is the standard one. Priest and Routley [39, 40] provide a nice discussion of the issue.

[17] For a discussion, see Priest [35].

[18] Depending on the details of the given logic, strong paraconsistentists sometimes collapse into dialetheic paraconsistentists. For discussion see Restall [42] and Beall and Restall [17].

to the current volume, is the third class: dialetheic paraconsistentists. What is important to note is that 'paraconsistency' and 'dialetheism' are *not* synonyms. Any rational version of the latter will require the former, but the converse seems not to hold.

Many of the contributions in this volume revolve around dialetheism. PRIEST's chapter argues that there are no good arguments against dialetheism.[19] Suppose that Priest's arguments are sound. Even so, an immediate question arises: Is there any reason to think that dialetheism is correct? Is there any reason to think that some contradictions are true? To that question I now (very briefly) turn.

5. TOWARDS A NON-EMPTY INTERSECTION

Let us suppose, as above, that *truth* and *falsity* are categories of sentences, with at least the constraint that $\neg\mathcal{A}$ is true if and only if $\mathcal{A}$ is false. Consider the following sentence (a 'Liar'):

» The first displayed sentence in s. 5 is false

Does that sentence go in category *truth* or in *falsity*? Given the way we use 'true', the first displayed sentence in s. 5 goes in *truth* only if it goes in *falsity*. But, given the way we use 'true', the first displayed sentence in s. 5 goes in *falsity* only if it goes in *truth*. What we seem to have, then, is a sentence that goes into the one category (truth) exactly if it goes into the other (falsity).

True contradiction? It depends. Suppose that *truth* and *falsity* are not exhaustive—that some sentences are in neither category, that there are 'truth value gaps'. Then we have no true contradiction, at least not via the first displayed sentence.

A question arises: When we say that the first displayed sentence is neither true nor false, what are we saying? One thing we are saying, it seems, is that the negation of the first displayed sentence is *not* false. But falsity is truth of negation, in which case we seem to be saying something of the form $\neg\neg\mathcal{A}$. (If T is our truth predicate and $\langle\mathcal{A}\rangle$ a name of $\mathcal{A}$, then we seem to be saying something of the form $\neg\mathsf{T}\langle\neg\mathcal{A}\rangle$, which is to say that $\neg\mathcal{A}$ is false, which seems to be equivalent to $\neg\neg\mathcal{A}$.) But, now, assuming Double Negation-Elimination, that entails $\mathcal{A}$. We seem to be back to the apparent true contradiction.

One natural suggestion is that we have at least two negations—one $\sim$ being a 'gap-closer', the other $\neg$ affording gaps. The idea is that we use the 'gap-closer' (sometimes called 'exclusion') when we say that the first displayed sentence in s. 5 is *not* false (or true). While that suggestion will avoid the problem above, it also

[19] Of course, PRIEST's contribution was written prior to the others in this volume. Debate will tell whether some of those considerations work against dialetheism.

returns us to the appearance of true contradiction:

The second displayed sentence in s. 5 is not true (or false)

It seems that the non-exhaustiveness of *truth* and *falsity* does little to avoid the apparent emergence of contradiction: The second displayed sentence seems to be true if and only if it is not. A simple lesson to draw is the dialetheic one: The second displayed sentence is *in the intersection* of both truth and falsity—or the intersection of truth and 'untruth' (if one adds that category to accommodate gaps).

Anyone familiar with contemporary work on the Liar will know that, in an effort to avoid 'true contradictions', many different non-dialetheic avenues have been pursued.[20] Some of the given avenues are ingenious attempts to avoid the apparent inconsistency, and most are mathematically or logically interesting frameworks for thinking about language. In the end, though, none of the given approaches are as simple as a dialetheic response, which simply accepts that the intersection of truth and falsity is non-empty. And given some suitable paraconsistent logic, the dialetheist may accept that some *but not all* contradictions are true—the non-empty intersection may be approached and enjoyed without explosive traffic.[21]

Simple or not, one might think, it seems downright irrational to accept that the intersection of truth and falsity is non-empty—that there are truths with true negations, that there are 'true contradictions' (even if they don't explode). Such a sentiment remains prominent—a residual vestige, perhaps, of the 'unassailable dogma' of (non-)contradiction. But it really is just dogma, at least as far as I can tell (and notwithstanding some of the contributions in this volume), but you (the reader) can judge for yourself.

One issue that should be emphasized is that nothing in dialetheism requires the existence of *observable* contradictions—true contradictions that have observable (but inconsistent) consequences. *That*, despite considerations to the contrary [7, 33], is difficult to understand. But one might, as some suggest,[22] restrict dialetheism to the purely semantic fragment of the language. In that case, the charge of 'irrationality' or even 'incredulous stares' are difficult to appreciate, as the

[20] For a discussion of contemporary approaches, see Beall [11, 12]. Priest [31] gives extended arguments against many such approaches, and also gives one of the earliest and most extended arguments for a dialetheic approach. Beall [10] presents arguments for a different (non-Priestly, as it were) version of dialetheism.

[21] Priest [31] has launched various arguments for dialetheism. The case from semantic paradox, by Priest's lights, is not as strong as the overall case from what he calls 'the inclosure schema' and 'principle of uniform solutions'[37]. Given that Priest's work is largely responsible for the 'spread of dialetheism' (slow as the spread may be), many of the chapters in this volume discuss a variety of Priest's arguments. My own thinking is that, regardless of 'inclosure' or the like, simplicity and preservation of naïve appearance is sufficient for accepting some version or other of dialetheism. But that too, in the pages to come, is challenged by various contributors. Zalta and Goldstein, for example, offer direct challenges by proposing alternative responses to various apparent inconsistencies. Armour-Garb discusses whether, and in what sense, dialetheism offers a solution to *paradox*.

[22] See the chapters by Beall and Mares.

only 'true contradictions' are grammatical residue (like the first or second displayed sentences) that carry no observational import. All that is claimed, at least on such restricted dialetheic positions, is that the intersection of truth and falsity contains various peculiar—but none the less grammatically inevitable—sentences that carry no observational consequences. Provided, as above, that a suitable paraconsistent logic is in place, there seems to be little to back worries of irrationality or instability or the like—little, again, beyond the dogma.

6. BEYOND THE SEMANTIC PARADOXES?

One would be misled to think that the *only* considerations towards true contradictions involve semantic paradoxes. Are there reasons to think that some contradictions, having nothing at all to do with the semantic paradoxes, are true? Debate will tell, but I briefly mention two considerations towards the possibility.[23]

Naïve Extensions

Priest [31] argues that the paradoxes of set theory, and in particular Russell's paradox, calls out for a dialetheic solution. Part of Priest's argument turns on his 'inclosure schema' and 'principle of uniform solutions' [37]. In effect, the argument is that Russell's paradox and the semantic paradoxes have the *same basic structure*—what Priest calls 'inclosure'—and, hence, ought to receive the same solution. While I am sympathetic with Priest's argument, I leave its details and merits to the reader.

By my lights, 'Russell's paradox' is ambiguous. On one hand, it denotes a type of paradox that arises in *set theory*, a discipline within mathematics. Sets were originally constructed within and for mathematics. If mathematics wishes to remain consistent, then Russell's *set*-theoretic paradox may be resolved as it has been—by stipulating it out (via axioms or the like).[24] Whether a set-theory is mathematically sufficient is governed by the pragmatic issue of whether it does the job—whether sets, so specified, do the trick for which they were constructed. In that respect, Russell's paradox may have a simple, consistent solution, at least for purposes of mathematics. And the same would go, of course, for mathematical versions of the Liar—stipulate them out, so long as the job is still fully achieved.

[23] One would likewise be misled to think that the following two points exhaust the considerations, or are even the strongest. Priest [37] covers a wide variety of other areas that arise, as he puts it, 'at the limits of thought and language'. Priest [31] also discusses the apparent inconsistency involved in *change, motion, legal contexts*, and much else.

[24] Arguments towards, and explorations of, *inconsistent mathematics*, may be found in Mortensen [29] (and references therein).

But there is another Russell's paradox, the paradox of (naïve) *extensions*, which arises not in the restricted confines of mathematics but in natural language. Semanticists and philosophers of language have long recognized the need for *extensions* of predicates (and expressions, in general). A look down the corridor reveals the mathematician's *sets*—and we have since been off running. The trouble is that there is no a priori reason to think that *sets* (the entities constructed within and for mathematics) will sufficiently play the role of extensions; indeed, there is reason to think otherwise. At least initially, with an aim on natural language, we want to have extensions for *every* predicate of the language. In particular, we want to have an extension not only for 'is a philosopher' and 'is a cat' but also for 'is an extension' and 'is not in its own extension' (i.e. 'χ 'is not in the extension of χ'). The simple idea, of course, is that our extension theory should not only be unrestricted but also should satisfy what seems plainly correct: that the denotation of a is in the extension of $\mathcal{F}$ iff $\ulcorner \mathcal{F}a \urcorner$ is true.[25] But having that calls for dialetheism, at least if one is to accept one's own theory.

I have not given an argument for true contradictions that arise from extensions, but it is an area in which true contradictions may well arise. While inconsistency in *set theory* can be resolved by axiomatizing away, the same is not clearly the case with respect to extensions. Extensions, unlike mathematical sets (at least on the picture I've suggested), are constrained not only by their role in our overall theories, but also by our 'intuitions' about them. Whether such a role or our given 'intuitions' yield true contradictions is something that, as always, debate will tell.

Borderline Cases

Another potential area in which true contradictions might arise is at the 'limits' of vagueness. Not a lot of work has been done on this topic, but a few considerations run as follows.[26]

So-called *tolerance conditionals* that appear in soritical paradoxes appear to be true. If b is a child at t_n, then b is a child at t_{n+1} (for some miniscule measure of time). Rejecting such conditionals, it seems, reveals an incompetence with respect to how the predicate 'is a child' (or any other vague predicate) is used. But the sorites paradox seems to challenge that appearance. Indeed, virtually all known approaches to the sorites reject at least one tolerance conditional, holding that it is

25 Likewise, of course, one wants to have an *extension* of 'is a truth', something that comprises all truths. The mathematicians' *sets*, as Grim [24, 25] argued, seem not to do the trick. All the more reason for an *extension* theory that does the trick.

26 Dominic Hyde [26] has advanced a paraconsistent, though not clearly dialetheic, approach to vagueness. For something closer to a dialetheic approach see Beall [6] and Beall and Colyvan [15, 16].

not rationally or competently assertable.[27] The trouble with such responses is that one none the less 'feels' that such conditionals *are* true.

One avenue towards resolving the issue is to recognize true contradictions at the 'limits' of vagueness. The suggestion, for example, is that all of the tolerance conditionals are true, but some of them are also false: they reside at the intersection of truth and falsity. In particular, the 'penumbra' is awash with true contradictions. A semantics that affords such an approach is covered below (LP, s. 7).

Of course, if vagueness affords true contradictions, then there may well be 'observable contradictions', and that may be a heavy cost to bear. But that issue deserves debate. In the end, it seems initially as reasonable to think that a 'vague language' is *overdetermined* as it is to think it *underdetermined.* But that issue, like others, is one that must here be left open.

Further discussion of dialetheism (both for and against), of course, may be found in the following chapters. For now, and for purposes of giving the reader a basic framework in which to think about some of the foregoing (and forthcoming) issues, I turn to a brief sketch of a common paraconsistent framework associated with dialetheism—Priest's 'logic of paradox', LP.

7. A SAMPLE PARACONSISTENT LOGIC

As above (s. 4), there are various standard approaches to paraconsistent semantics. Because of its 'classical' appearances (and, hence, familiarity), and also its historical tie to dialetheism, the focus here will be on a basic many-valued, truth-functional approach. The logic typically associated with dialetheism is Priest's 'logic of paradox', LP [30]. For purposes of generality, I present FDE but highlight LP in due course.

Propositional Semantics

The syntax is that of classical logic. The semantics arises by letting interpretations be functions ν from sentences into $\mathcal{V} = \wp(\{1, 0\})$. Hence, where $\mathcal{A}$ is any sentence, $\nu(\mathcal{A} = \{1\}, \nu(\mathcal{A}) = \{0\}, \nu(\mathcal{A}) = \{1, 0\}$, or $\nu(\mathcal{A}) = \emptyset$. Given that $\nu(\mathcal{A})$ is a set (comprising either 1, 0, or nothing), we may (by way of informal interpretation) say that $1 \in \nu(\mathcal{A})$ iff $\mathcal{A}$ is (at least) true under ν, and $0 \in \nu(\mathcal{A})$ iff $\mathcal{A}$ is (at least) false under ν. In the case where $\nu(\mathcal{A}) = \emptyset$, we may (informally) say that $\mathcal{A}$ is neither true nor false (under ν); and when $1 \in \nu(\mathcal{A})$ and $0 \in \nu(\mathcal{A})$, we may (informally) say that $\mathcal{A}$ is both true and false (under ν).

27 For recent work on the sorites, see Beall [9] and the references therein. (That volume also contains recent work on various semantic paradoxes.)

$\mathcal{D}$, our designated values, comprises $\{1\}$ and $\{1, 0\}$. (Intuitively, and informally, we designate all and only those sentences that are 'at least true'.)

We say that an interpretation ν is *admissible* just in case it 'obeys' the following clauses:[28]

» $1 \in \nu(\neg\mathcal{A})$ iff $0 \in \nu(\mathcal{A})$

» $0 \in \nu(\neg\mathcal{A})$ iff $1 \in \nu(\mathcal{A})$

» $1 \in \nu(\mathcal{A} \wedge \mathcal{B})$ iff $1 \in \nu(\mathcal{A})$ and $1 \in \nu(\mathcal{B})$

» $0 \in \nu(\mathcal{A} \wedge \mathcal{B})$ iff $0 \in \nu(\mathcal{A})$ or $0 \in \nu(\mathcal{B})$

Logical consequence (semantic consequence) is defined as 'truth preservation' over all (admissible) interpretations, that is, if every premise in Σ is at least true, then so too is $\mathcal{A}$:

» $\Sigma \Vdash \mathcal{A}$ iff $\nu(\mathcal{A}) \in \mathcal{D}$ if $\nu(\mathcal{B}) \in \mathcal{D}$, for all $\mathcal{B}$ in Σ

A sentence $\mathcal{A}$ is *valid* (a tautology, logical truth) exactly if $\emptyset \Vdash \mathcal{A}$.

Remarks

The foregoing semantics yields the propositional language of FDE (first degree entailment) [1, 2]. There are a few notable features of the current semantics.

» There are no valid sentences: Just consider the admissible interpretation according to which every sentence is neither true nor false. (Compare Kleene's 'strong' semantics K_3.)

» Suppose that we restrict the (admissible) interpretations to those interpretations the range of which is $\wp(\{1, 0\})-\{\{1, 0\}\}$. In that case, we have K_3, a simple 'gappy' semantics that is *not* paraconsistent.

» Suppose that we restrict the (admissible) interpretations to those interpretations the range of which is $\wp(\{1, 0\})-\{\emptyset\}$. In that case, we have LP, a simple 'glutty' semantics which *is* paraconsistent. As one can easily show, the valid sentences of LP and those of classical logic are precisely the same. (The consequence relation, of course, is different: LP-consequence is weaker, since it is not explosive.)

» Suppose that we restrict the (admissible) interpretations to those interpretations the range of which is $\wp(\{1, 0\})-\{\{1, 0\}\} \cup \{\emptyset\}$. In that case, we have classical semantics, which admits neither 'gluts' nor 'gaps' and is explosive.

[28] Disjunction $\vee$ and the hook $\supset$ (the 'material conditional') are defined in the usual way.

Quantification

The syntax, as in the propositional case, is that of classical (predicate) logic. Algebraic techniques for extending a many-valued propositional language to a quantified one are available; however, a straightforward, and perhaps more familiar, technique is available in the (non-algebraic) current case.

We let an interpretation be a pair $\langle \mathcal{O}, \delta \rangle$, where $\mathcal{O}$ is a non-empty set of objects (the domain of quantification) and δ a function that does two things:[29]

- δ maps the constants into $\mathcal{O}$
- δ maps every n-ary predicate $\mathcal{P}^n$ into a *pair* $\langle \mathcal{E}_{\mathcal{P}^n}, \mathcal{A}_{\mathcal{P}^n} \rangle$, where $\mathcal{E}_{\mathcal{P}^n} \subseteq \mathcal{O}^n$ and $\mathcal{A}_{\mathcal{P}^n} \subseteq \mathcal{O}^n$

$\mathcal{E}_{\mathcal{P}^n}$ is said to be the *extension* of $\mathcal{P}^n$ and $\mathcal{A}_{\mathcal{P}^n}$ the *anti-extension.* (The extension of $\mathcal{P}^n$, informally, comprises all the objects of which $\mathcal{P}^n$ is at least true, and the anti-extension the objects of which $\mathcal{P}^n$ is at least false.)

Atomic sentences are assigned 'truth values' (elements of ν) according to the familiar clauses:

- $1 \in \nu(\mathcal{P}^n c_1, \ldots, c_n)$ iff $\langle \delta(c_1), \ldots, \delta(c_n) \rangle \in \mathcal{E}_{\mathcal{P}^n}$
- $0 \in \nu(\mathcal{P}^n c_1, \ldots, c_n)$ iff $\langle \delta(c_1), \ldots, \delta(c_n) \rangle \in \mathcal{A}_{\mathcal{P}^n}$

Non-quantified compound sentences, in turn, are assigned values as per the propositional case (negation, conjunction, and, derivatively, disjunction, material implication, etc.). The clauses for quantifiers run thus:[30]

- $1 \in \nu(\forall \chi \mathcal{A})$ iff $1 \in \nu(\mathcal{A}(\chi/c))$, for every $c \in \mathcal{O}$
- $0 \in \nu(\forall \chi \mathcal{A})$ iff $0 \in \nu(\mathcal{A}(\chi/c))$, for some $c \in \mathcal{O}$
- $1 \in \nu(\exists \chi \mathcal{A})$ iff $1 \in \nu(\mathcal{A}(\chi/c))$, for some $c \in \mathcal{O}$
- $0 \in \nu(\exists \chi \mathcal{A})$ iff $0 \in \nu(\mathcal{A}(\chi/c))$, for every $c \in \mathcal{O}$

Logical consequence is defined as per usual: 'truth preservation' over all (admissible) interpretations.

Remarks

Not surprisingly, classical semantics (and, similarly, strong Kleene 'gappy' semantics) may be 'regained' by imposing appropriate constraints on the foregoing semantics, and in particular on what counts as an admissible interpretation. Example: By imposing the constraint that $\mathcal{E}_{P^n} \cup \mathcal{A}_{P^n} = \mathcal{O}^n$ and $\mathcal{E}_{P^n} \cap \mathcal{A}_{P^n} = \emptyset$

[29] For simplicity, assume that every element of $\mathcal{O}$ has a name, and in particular that elements of $\mathcal{O}$ name themselves and, thus, function as constants.

[30] One of the quantifiers is taken to be defined (per usual) but, despite redundancy, clauses for both quantifiers are given here. $\mathcal{A}(\chi/c)$ is $\mathcal{A}$ with every free occurrence of χ replaced by c. (Usual caveats about bondage are in place! And recall that $c \in \mathcal{O}$ serves as a name of itself.)

(for any predicate P^n), one 'regains' classical semantics. As in the propositional case, the upshot is that any classical (first-order) interpretation is a (first-order) FDE-interpretation, and so the former is a (proper) extension of the latter.

The foregoing semantics can be (and have been) augmented to include function symbols, identity, and modal operators (and also extended to second-order). For present purposes, I leave those extensions aside.[31]

8. BUT WHAT OF THE APPARENT LOSS?

Suppose that for purposes of adopting dialetheism we accept LP. We may then enjoy a simple response to the intersection of truth and falsity: it is non-empty, but no explosive traffic ensues.

But what about the apparent loss? We avoid explosion, to be sure; however, we thereby lose Disjunctive Syllogism (DS)—the inference from $\mathcal{A} \vee \mathcal{B}$ and $\neg\mathcal{A}$ to $\mathcal{B}$.[32] But we reason with DS all the time, and it is not clear whether we could do without it. If not, the 'gain' of simple dialetheism is too expensive to bear.

The concern is an important and natural one, one that frequently emerges in early discussion of dialetheism. I will not dwell on the issue here, but it is important to say something on the matter.[33]

In the first instance, the response is (of course) that there is no genuine loss. If dialetheism is true and LP the appropriate logic, then DS was never really truth-preserving. (One cannot lose something that was not there.) Moreover, if (as it appears) Liar-like sentences are the only root of the invalidity, it is not surprising that we would think DS to be valid, since Liars are easy to overlook.

There is more to say. In particular, it is not abundantly clear that we really do employ DS in our standard reasoning, as opposed to a closely related 'rule of inference'. The dialetheist, as Priest [31] emphasizes, is free to follow the rationality-version of 'Disjunctive Syllogism':

» If one accepts $\mathcal{A} \vee \mathcal{B}$ and one rejects $\mathcal{A}$, then one ought *rationally* accept $\mathcal{B}$

Provided that acceptance and rejection are exclusive (though they needn't be exhaustive), the 'rationality version' is a principle by which one can regain the

[31] See Priest [34, 35] for details (and also a suitable proof theory). Littmann and Simmons's chapter raises interesting issues involving descriptions in a dialetheic setting.

[32] The reader familiar with 'material modus ponens' will recognize that that 'also' is lost—as it is little more than DS in disguise. Accordingly, a detachable conditional must be added to the language. A variety of conditionals is available. Priest [31] contains discussion, and recent work on 'restricted quantification' by Beall, Brady, Hazen, Priest, and Restall [14] introduces a new option. Because of lack of space, I leave that (admittedly important) topic aside.

[33] And, of course, a paraconsistent logic in which DS is preserved but some other 'classic' inference is gone is one for which precisely the same issue arises. There is nothing peculiar about DS, except that its 'loss' is often associated with dialetheism.

reasoning that often passes for (the invalid) DS. If that is right, then the 'loss' of DS seems not to be a great loss, after all.[34]

Finally, it is important to note that a dialetheist has no reason to reject consistency as a default assumption, or as a high theoretical virtue, in general. That some contradictions are true does not imply that most contradictions are true—especially if such true contradictions turn out to be only the peculiar paradoxical sentences. (Even if other sorts of sentences, beyond the paradoxical ones, yield true contradictions, the point still applies.) All that the dialetheist requires is that the default aim of consistency is just that: it is *default*, not absolute.[35]

9. BUT WHAT OF TRUTH?

Beyond the concern about 'losing' DS, there are (regrettably) few other articulated objections against dialetheism. The few standard worries—epistemic, belief revision, and the like—are discussed in PRIEST's chapter, and I leave them to that essay.[36] I close by mentioning one topic that philosophers tend to worry about when the notion of 'true contradiction' is raised: Truth.[37]

Some philosophers might think that there is something in the 'nature' of truth that rules out the existence of true contradictions. But on reflection, the thought seems not to pan out. Consider, for example, the two main approaches to truth: correspondence and deflationism. (I don't say the *only* two, but the two main contenders.) The latter, as Priest [36], BEALL, and Beall and Armour-Garb [4, 13] have argued, seems to yield dialetheism quite naturally. After all, there is no 'nature' to bar the grammatically inexorable true contradictions; there are simple rules of dis-quotation and en-quotation (or simply inter-substitution)—and that's it. Deflationists might well seek to avoid true contradictions, but (again) one wonders why such avoidance is sought—especially when, as it appears, the avoidance-procedures make for a much more complicated position.

[34] SHAPIRO's chapter challenges the current move to some extent, in as much as it challenges the dialetheist's ability to give a coherent notion of *exclusion*. I leave the reader to weigh the merits of Shapiro's arguments against the proposed move. (I should also point out that, as far as I can see, Shapiro's chief objections may not affect a version of dialetheism underwritten by a logic other than LP (or, for that matter, FDE). For one such alternative approach, see Beall [10].)

[35] See the appendix of BEALL's chapter for brief discussion and references on 'default consistency'.

[36] There are other, more technical worries that I will omit here. One such is Curry's paradox, but that depends on which conditional is in play—a topic that I have omitted here. (A dialetheic response to the 'material conditional' version of Curry is precisely the same as the general response to Liars. A detachable conditional, as above, is where the issue arises. See [31] for discussion.) A similar issue concerns so-called Boolean negation. RESTALL's chapter, as with BRADY's, PRIEST's, and SAINSBURY's, touch on that issue.

[37] Many of the contributions in this volume presuppose one stance or another on truth, but the chapters by GARFIELD, COGBURN, and TENNANT have direct bearing on the topic, as does KROON's. BEALL's chapter specifically focuses on (one conception of) truth.

More interesting are concerns that arise from correspondence. While there remains no clear account of 'correspondence', the basic idea is clear enough. The idea (not formulated as such by all 'robust theorists', but common enough for present purposes) is that any truth has a truth-maker—that any truth is 'made true' by 'the facts', by some actual 'something' in the world without which a putative truth would fail to correspond and, hence, fail to be a truth. Now suppose, as per dialetheism, that there are truths of the form $\mathcal{A} \wedge \neg\mathcal{A}$. Such a truth would require truth-makers for both $\mathcal{A}$ and $\neg\mathcal{A}$. But how could that be?

The worry, in the end, is not substantial. Whether correspondence is the right approach to truth remains an open (and much debated) question [22]. Suppose, though, that correspondence is the right approach, and that each truth requires a truth-maker. What, exactly, is the worry about having truth-makers for both $\mathcal{A}$ and $\neg\mathcal{A}$? On the surface, no particular problem presents itself, at least not one that is peculiar to dialetheism. To be sure, dialetheism requires that there be 'negative truth-makers', since at least one 'negative truth' is true if both $\mathcal{A}$ and $\neg\mathcal{A}$ are true. But that is a general problem for correspondence, not one peculiar to dialetheism. Moreover, the problem of accommodating 'negative truths' is not particularly difficult; there are standard models available, due to van Fraassen [46], Barwise [5], and others.[38] The worry, as said, seems not to be substantial—at least pending further details.

10. AT THE CROSSROADS: CLOSING REMARKS

Unfortunately, and despite the enormous activity in paraconsistent logic over the last thirty years, there has been little debate centred on non-contradiction—or, at least, little by way of *defense*. Perhaps many have echoed Łukasiewicz in thinking that, while Aristotle's arguments are (at best) insubstantial, Simple (Non-)Contradiction, or perhaps Rationale (Non-)Contradiction, are 'unassailable dogmas' that need only be entrenched, as opposed to defended.[39] Such a thought is philosophically suspect. The incredulous stare was an insufficient 'reply' to modal realism; and it is an insufficient 'reply' to dialetheism.

The hope behind the current volume is that debate may move forward, and that the attitude of unassailable dogma swiftly slides into the past. The intersection is before you; the question is whether it is empty.[40]

[38] Note that van Fraassen's given work was not intended to yield 'negative facts', but it yields a suitable framework for them none the less. For further discussion and details of suitable frameworks, see Beall [8].

[39] What is interesting is that Łukasiewicz's student Jaskowski [27] was an early pioneer of contemporary paraconsistent logic.

[40] I am grateful to Brad Armour-Garb, Mark Colyvan, and Dave Ripley for discussion and comments. Special thanks to Graham Priest and Greg Restall for discussion over the last few years, especially early on in Oz, where this volume was conceived—back in 1999! Thanks, finally, to Katrina Higgins for her support, and also for her patience with this and related projects.

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AQ: Please update references [11], [12], [14] and [17].

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