Where the Paths Meet: Remarks on Truth and Paradox*

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The study of truth is often seen as running on two separate paths: the *nature* path and the *logic* path. The former concerns metaphysical questions about the 'nature', if any, of truth. The latter concerns itself largely with logic, particularly logical issues arising from the truth-theoretic paradoxes.

Where, if at all, do these two paths meet? It may seem, and it is all too often assumed, that they do not meet, or at best touch in only incidental ways. It is often assumed that work on the metaphysics of truth need not pay much attention to issues of paradox and logic; and it is likewise assumed that work on paradox is independent of the larger issues of metaphysics. Philosophical work on truth often includes a footnote anticipating some resolution of the paradox, but otherwise tends to take no note of it. Likewise, logical work on truth tends to have little to say about metaphysical presuppositions, and simply articulates formal theories, whose strength may be measured, and whose properties may be discussed. In practice, the paths go their own ways.

Our aim in this paper is somewhat modest. We seek to illustrate one point of intersection between the paths. Even so, our aim is not completely modest, as the point of intersection is a notable one that often goes unnoticed. We argue that the 'nature' path impacts the logic path in a fairly direct way. What one can and must say about the logic of truth is influenced, or even in some cases determined, by what one says about the metaphysical nature of truth. In particular, when it comes to saying what the well-known Liar paradox teaches us about truth, background conceptions — views on 'nature' — play a significant role in constraining what can be said.

This paper, in rough outline, first sets out some representative 'nature' views, followed by the 'logic' issues (viz., paradox), and turns to responses to the Liar paradox. What we hope to illustrate is the fairly direct way in which the background 'nature' views constrain — if not dictate — responses to the main problem on the 'logic' path. (We also think that the point goes further, particularly concerning the relevance and appropriate responses to 'Liar's revenge'. We will

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return to this briefly in the concluding §4.) In §1, we discuss two conceptions of truth; one in the spirit of contemporary deflationism, and the other in the spirit of the correspondence theory of truth. The given conceptions (or 'views') serve as our representatives of the nature path. In §2, we briefly present issues relevant to the 'logic' path, and particularly the Liar paradox. In §3 we show that our two views of the nature of truth lead to strikingly different options for how the paradox — how questions of logic — may be addressed. We show this by taking each view of the nature of truth in turn, and examining the range of options for resolving the Liar they allow. We close in §4 by considering one further point where the two paths meet, related to how to understand 'revenge paradoxes'.

1 Nature: Two Conceptions of Truth

We distinguish two paths in the study of truth: the nature path and the logic path. The nature path is traditionally one of the mainstays of metaphysics (and perhaps epistemology as well). It was walked, for instance, by the great theories of truth of the early 20th century: the correspondence theory of truth, the coherence theory of truth, and the pragmatist theories of truth. The same may be said of more recent philosophical views of truth, including, on a more skeptical note, deflationist theories of truth. The logic path is usually thought of as studying the *formal* properties of truth, and in particular, studying them with the goal of resolving the well-known truth-theoretic paradoxes such as the Liar paradox.

In this section, we articulate two ways of approaching the metaphysics of truth — two ways of following the nature path. One is a 'deflationary' conception of truth, and the other 'correspondence-like'. Many approaches to the metaphysics of truth have been developed over time, and we do not attempt to survey them all. Instead, we briefly discuss these two accounts, which we think are fairly representative of the main trends in the metaphysics of truth, and also, fairly familiar.[1] Once we have presented these two ways down the nature path, we turn to the logic path, and then to how the two meet. Before launching into our two representative views, however, we pause to explore a little further what the nature path in the study of truth seeks to accomplish.

The touchstones for current philosophical thinking about truth are the theories developed in the early 20th century, such as the classic coherence and correspondence theories of truth. It is not easy to give an historically accurate representation of either of these ideas. But for our purposes, it will suffice to make use of the crude slogans that go with such theories. The correspondence theory may be crystallized in the view that truth is a correspondence relation between a truth bearer (e.g., a proposition) and a truth maker (e.g., a fact). The correspondence relation is typically some sort of mirroring or representing

[1] We also confess to having strong bias towards the given accounts, with each author favoring a different one.

relation between the two. In contrast, a coherence theory holds that a truth bearer is true if it is part of an appropriate coherent set of such truth bearers.[2]

Caricatures though these slogans may be, they are enough to see what the main goal of theories of this sort is. They seek to answer the *nature question*: what sort of property is truth, and what is it that makes something true. As such, they have no particular interest in the *extent question*: what is the range of truths.[3] We take *philosophical theories of truth* to be theories that answer the nature question. Hence, we call the path that pursues traditional philosophical questions about truth *the nature path*.

Contemporary discussion of the nature question has focused on whether there is really any such thing as a philosophically substantial nature to truth at all. Deflationists of many different stripes argue there is not. Descendants of the traditional views, especially the correspondence theory, hold that there is, and seek to elucidate it. The 'semantic' view we discuss below seeks to do so in a way that is less encumbered by the metaphysics of the early 20th century — especially, the metaphysics of facts — but still captures the core of the correspondence idea.

We shall thus present two representative views, which we believe give a good sample of the options for the nature path. The first, which we call the *semantic* view of truth, is a representative of a substantial and correspondence-inspired answer to the nature question. The second, which we call the *transparent* view of truth, is a form of deflationism, taking a skeptical stance towards the nature question. Obviously, these by no means exhaust the options, or even the options that have received strong defenses in recent years, but they give us typical examples of the main options, and so allow us to compare how philosophical accounts of the nature of truth relate to the formal or logical properties of truth.

Parenthetical remark. One issue that was often hotly debated in the classical nature literature was that of what the primary bearers of truth are.[4] For purposes of this essay, we take a rather casual view towards this question. We will talk of sentences as the bearers of truth; particularly, sentences of an *interpreted* language. At some points, it will be crucial that our sentences be interpreted, and have rich semantic properties. When we make reference to formal theo-

[2]For a survey of these ideas, and pointers to the literature, see Glanzberg [Gla06b]. The correspondence theory is associated with work of Moore (e.g. [Moo53]) and Russell (e.g. [Rus10, Rus12, Rus56]), though their actual views vary over time and are not faithfully captured by the correspondence slogan. (Indeed, both started off rejecting the correspondence theory in their earliest work.) Notable more recent defenses include Austin [Aus50]. The coherence theory is associated with the British idealist tradition that was attacked by the early Russell and Moore, notably Joachim [Joa06], and later Blanshard [Bla39]. (Whether Bradley should be read as holding a coherence theory of truth has become a point of scholarly debate, as in Baldwin [Bal91].) For a discussion of the coherence theory, see Walker [Wal89].

[3]Some philosophers, notably Dummett (e.g [Dum59, Dum76]) approach both nature and extent questions together.

[4]For instance, the question of whether there are propositions, and whether they can serve as truth bearers, was crucial to Russell and Moore's turn from the identity theory of truth to the correspondence theory.

ries, sentences are the convenient elements with which to work. But it would not matter in any philosophically important way if we were to replace talk of interpreted sentences with talk of utterances which deploy them, or propositions whose contents they express, or any other favored bearers of truth. *End parenthetical.*

1.1 Semantic Truth

The first view of the nature of truth we sketch is what we call the *semantic* view of truth. We see it as a descendant of the classical correspondence theory, and a representative of that idea in the current debate.

The view we sketch takes truth to be a key semantic property. This is a familiar idea. It is the starting point to many projects in formal semantics, which seek to describe the semantic properties of sentences in terms of assignments of truth values (or more generally, truth conditions). It is also the starting point of any model-theory-based approach to logic. Just what sorts of semantic values may be assigned, and what is done with them, differ from project to project, but that there are theoretically significant semantic values to be assigned to sentences, and that one of them (at least) counts as a truth value, is a common idea in logic and semantics.[5] This idea is familiar, but it is also familiar to see it contested. As our goal is to present a representative approach to truth, we will not pause to defend it, so much as see how the familiar idea leads to a view of the nature of truth.

Theories of semantics or model theory of this sort use a 'truth value', but it is typically a rather abstract matter just what a 'truth value' in such a theory is. It is, for most purposes, an arbitrarily chosen object, often the number 1. What is important is the role that assigning that object to sentences plays in a semantic or logical theory. The semantic view of truth takes the next step, and holds that for the right theory, a theory of this semantic value is indeed a theory of the nature of truth. Truth is this fundamental semantic property, and the nature of truth is revealed by the nature of the underlying semantics. The truth predicate, which expresses truth, has as its main job to *report* this status. The 'nature' of the concept expressed by a truth predicate Tr is the nature of the underlying semantic property that the truth predicate reports.

We have suggested that the semantic view of truth (as we use the term) is the heir to the classical correspondence view of truth. Put in such abstract terms, it may not be obvious why, but it becomes more clear if we think of how the truth values of sentences are determined, and how this is reflected in semantic theories. Let us assume, as is fairly widely done, a broadly referential picture. Terms in our sentences denote individuals. Predicates one way or another pick out properties (or otherwise acquire satisfaction conditions). A simple atomic sentence gets the value 1 ($\mathtt{t}$ or whatever the theory posits) just in case the individual bears the property. A semantic theory in the truth-conditional vein

[5] In semantics, one can see any work in the truth-theoretic tradition, e.g. Heim and Kratzer [HK98] or Larson and Segal [LS95]. In logic, any book on model theory will suffice. To see such ideas at work in a range of logics, see Beall and van Fraassen [BvF03].

tells us how a sentence gets is semantic value in virtue of the referents of its parts. Our semantic view of truth holds that this is in fact telling us what it is for the sentence to be *true*. But here, we see the correspondence idea at work. What determines whether a sentence is true is what in the world its parts pick out, and whether they combine as the sentence says.

The semantic view of the nature of truth does not rest on a metaphysics of facts, as many forms of the classical correspondence theory did.[6] Rather, determinate truth values are built up from the referents of the right parts of a sentence. Whereas a classical correspondence theory would look for some sort of *mirroring* between a truth bearer and a truth maker, like a structural correspondence between a fact and a proposition, the semantic theory rather looks to the semantic properties of the right parts of a sentence, and builds up a truth value based on them for the sentence as a whole, according to principles of semantic composition. Reference for parts of sentences, plus semantic composition, replaces correspondence.

Though the metaphysics of facts is not required, this is an account of truth in terms of relations between sentences and the world. Especially if we take the route envisaged by Field [Fie72], which seeks to spell out the basic notions like reference on which the semantic view is built, this view shows truth to be a metaphysically non-trivial relation between truth bearers and the world. The relation is no longer one of a truth bearer to single truth maker, but it remains a substantial word-to-world relation, which we may think of as correspondence, or rather, all the correspondence we need.[7] The semantic view is thus, we say, a just heir to the correspondence theory. It can likewise support the questions of realism and idealism that where the focus of the correspondence theory. It seeks an answer to the nature question for truth which follows the lead the correspondence theory set down.[8]

As we use the term 'semantic truth', its key idea is that the predicate Tr reports a semantic property of sentences. Notoriously, Tarski [Tar44] talked about a 'semantic conception of truth'. We are not at all sure if our semantic truth is what Tarski had in mind, and his own claims about the semantic conception are not clear on the issue. Regardless, we have clearly borrowed heavily from Tarski (especially Tarski [Tar35]) in formulating the semantic view.[9] We shall use our notion of semantic truth as a representative of a substantial correspondence-inspired view of truth.

[6]This is not to say that the 'semantic view', as we use the term, cannot rest on a metaphysics of facts. See Taylor [Tay76] for one example, as well as Barwise and Perry [BP86], and Armstrong [Arm97].

[7]There are contemporary views that put much more weight on the existence of the right object to make a sentence true, such as the 'truth maker' theories discussed by Armstrong [Arm97], Fox [Fox87], Mulligan, Simons, and Smith [MSS84], and Parsons [Par99].

[8]There are classical roots for this sort of theory. It echoes some ideas tried out by Russell in the so-called 'multiple relation theory' (e.g. Russell [Rus21]). Perhaps more tendentiously, we believe that it is close to what Ramsey had in mind in [Ram27] (in spite of Ramsey usually being classified as a deflationist). (An ongoing project by Nate Smith is developing the point about Ramsey in great detail.)

[9]That Tarski's work might be pressed into the service of a correspondence-like view was also noted by Davidson [Dav69].

1.2 Transparent Truth

So far, we have briefly described one approach to the nature question: our correspondence-inspired semantic view of truth. In the current debate, perhaps the main opposition to views like this one are deflationist positions that hold that there is not really any substantial answer to the nature question at all. Our next view of the nature of truth, which we call the *transparent* view of truth, is a representative of this sort of approach.

There are many forms of deflationism about truth to be found. Transparent truth takes its inspiration from disquotationalist theories. According to these theories, there is no substantial answer to the nature question, as the nature question, though grammatical, asks after something that does not exist. Truth, according to these views, is not a property with a fundamental nature; it is simply an expressive device that allows us to express certain things that would be difficult or in-practice impossible without it. As is commonly noted, for instance, truth allows the expression of generalizations along the lines of 'Everything Max says is true', and allows for affirmation of claims we cannot repeat, along the lines of 'The next thing Agnes says will be true'. Truth is a device for making claims like this, and nothing more; it is thus not in any interesting way a property whose 'nature' needs to be elucidated.

Notionally, we may think of such an expressive device as *added* to a language. Adding the device increases its expressive power, but not by adding to its 'ideology' (as Quine [Qui51] would put it). The crucial property that allows truth to play this role is what we call *transparency.* A predicate $Tr(x)$ is *transparent* if it is see-through over the whole language: $Tr(\ulcorner\phi\urcorner)$ and ϕ are intersubstitutable in all (non-opaque) contexts, for all ϕ in the language.[10] Transparency is the key property that allows truth to affect expressive power. It does so by supporting inferences from claims of truth to other claims. For instance, we can extract the content of 'everything Max says is true' by first identifying what Max says, and next applying the transparency property. A transparent predicate is a useful way to allow generalization over sentences, and to extract content from those generalizations.[11]

The transparent view of truth has it that truth is simply a transparent predicate, and so can perform these expressive functions. There is nothing more to it. As we mentioned, it is useful to think of a transparent truth predicate as having been added to a language, to add to its expressive power. But importantly, a transparent truth predicate is defined to be *fully* transparent; it allows

[10] At the very least, such intersubstitutability amounts to bi-implication. So, the transparency of $Tr(x)$ amounts to the following. Where β is any sentence in which α occurs, the result of substituting $Tr(\ulcorner\alpha\urcorner)$ for any occurrence of α in β implies β and vice versa.

[11] The disquotationalist variety of deflationism stems from Leeds [Lee78] and Quine [Qui70]; the particular case of the transparent view is discussed by Beall [Bea05, Bea08d] and Field [Fie86, Fie94]. Other varieties of deflationism include the minimalism of Horwich [Hor90], and various forms of the redundancy theory, such as that of Strawson [Str50] and the view often attributed (we think mistakenly) to Ramsey [Ram27]. The latter is developed by Grover, Camp, and Belnap [GKB75]. For discussion of deflationary truth in general, see the chapters in Armour-Garb & Beall [AGB05]

intersubstitutability for all sentences of the language, including those in which the truth predicate figures. This is the defining feature that allows the truth predicate to play its expressive role, and so, to the transparent view, it is the defining feature of truth.

The transparent view of truth will be our representative deflationist approach, and our second representative philosophical approach to truth. Each of our representative views takes a stand on the nature question. We thus have one substantial correspondence-like view of truth, and one deflationary view, to represent the nature path to truth.[12]

2 Background on Logic and Paradox

In order to discuss the 'logic' path, and where our two paths meet, we need to set up a bit of background. This section provides the needed background on logic, formal theories of truth, and the Liar paradox. Where the two paths come together is discussed in the following §3.

2.1 Background on Logic

In discussing logics, our main tool will be that of interpreted formal languages. For our purposes, an *interpreted formal language* (or just a 'language') $\mathcal{L}$ is a triple $\langle \mathtt{L}, \mathcal{M}, \sigma \rangle$, where $\mathtt{L}$ is the syntax, $\mathcal{M}$ a 'model' or 'interpretation', and σ a 'valuation scheme' (or 'semantic value scheme'). We do not worry much about syntax here, though from time to time we are careful to note whether a given language contains a truth predicate Tr in its syntax. Unless otherwise noted, we assume the familiar syntax of first order languages.

Elements of interpreted, formal languages to which we do pay attention are models and valuation schemes. A model $\mathcal{M}$ provides interpretations of the non-logical symbols (names, predicates, and if need be, function symbols). A model has a domain of objects, and names are assigned these as values. To allow for a suitable range of options for dealing with paradox, we are more generous with the interpretations of predicates (and sentences) than might be standard. A predicate P will be assigned a pair of sets of (n-tuples of) elements of the domain, written $\langle P^+, P^- \rangle$. P^+ is the *extension* of P in $\mathcal{M}$, and P^- is the *antiextension*. Importantly, P can be given a partial interpretation, or an overlapping or 'glutty' interpretation. If $\mathcal{D}$ is the domain of $\mathcal{M}$, we do not generally require either of the following.

- Exclusion constraint: $P^+ \cap P^- = \emptyset$.
- Exhaustion constraint: $P^+ \cup P^- = \mathcal{D}^n$.

Classical models satisfy both the Exhaustion and Exclusion constraints, but we consider logics where they do not hold.[13]

[12] For more comparisons between correspondence and deflationary views, see David [Dav94].

[13] Hence, for classical languages, it is common to dispense with P^-, as $P^- = \mathcal{D}^n \setminus P^+$, where $X \setminus Y$ is the complement of Y in X (i.e, everything in X that is not in Y).

With neither Exhaustion or Exclusion guaranteed, we have to be more careful about how we work with values of sentences. This is where a valuation scheme comes into the picture. The job of a valuation scheme σ, relative to a set $\mathcal{V}$ of so-called 'semantic values', is to give a definition of *semantic value* for sentences of `L`, from the interpretations of non-logical expressions in a model $\mathcal{M}$. Furthermore, having a valuation scheme allows us to describe notions of validity and consequence, as we allow models to vary. We will illustrate with three important examples: a *Classical* language, a *Strong Kleene* language, and a *Logic of Paradox* language.

First, a Classical language. Fix a model $\mathcal{M}$ obeying the Exhaustion and Exclusion constraints. The *Classical valuation scheme* τ is defined on a set of semantic values $\mathcal{V} = \{1, 0\}$. We use $|\phi|_{\mathcal{M}}$ for the semantic value of ϕ relative to $\mathcal{M}$. The main clause of the Classical valuation scheme τ is the following.

$$|P(t_1, \ldots, t_n)|_{\mathcal{M}} = \begin{cases} 1 & \text{if } \langle |t_1|_{\mathcal{M}}, \ldots, |t_n|_{\mathcal{M}} \rangle \in P^+ \\ 0 & \text{if } \langle |t_1|_{\mathcal{M}}, \ldots, |t_n|_{\mathcal{M}} \rangle \in P^- \end{cases}$$

Clauses for Boolean connectives and quantifiers are defined in the usual way.

Interpreted languages give us logical notions in the following way. For a fixed syntax and valuation scheme, we can vary the model, and in doing so, ask about logical truth and consequence. In the Classical case, we have the following. We say that a set Γ of sentences *classically implies* a sentence ϕ if there is no classical model in which τ assigns every member of Γ the value 1, and ϕ the value 0.

The apparatus of interpreted languages allows us to explore many non-classical options as well. We mention two examples, beginning with an example of a 'paracomplete logic' based on the familiar *Strong Kleene language*. (For more on the 'paracomplete' and 'paraconsistent' terminology, see §3.) Strong Kleene models are just like classical models except that they drop the Exhaustion constraint on predicates (but keep the Exclusion constraint).

The Strong Kleene valuation scheme κ expands the set $\mathcal{V}$ of 'semantic values' to $\{1, \frac{1}{2}, 0\}$. The clause for atomic sentences is modified as follows.[14]

$$|P(t_1, \ldots, t_n)|_{\mathcal{M}} = \begin{cases} 1 & \text{if } \langle |t_1|_{\mathcal{M}}(t_1), \ldots, |t_n|_{\mathcal{M}} \rangle \in P^+ \setminus P^- \\ 0 & \text{if } \langle |t_1|_{\mathcal{M}}, \ldots, |t_n|_{\mathcal{M}} \rangle \in P^- \setminus P^+ \\ \frac{1}{2} & \text{otherwise.} \end{cases}$$

Three-valued logical connectives may be defined by the following rules. For negation: $|\neg\phi|_{\mathcal{M}} = 1 - |\phi|_{\mathcal{M}}$. For disjunction: $|\phi \vee \psi|_{\mathcal{M}} = \max\{|\phi|_{\mathcal{M}}, |\psi|_{\mathcal{M}}\}$. (These rules work equally well for the Classical τ or the Strong Kleene κ, but the range of values involved is different for each.)

We define *Strong Kleene consequence* — or K_3 consequence — much as before: ϕ is K_3 *implied by* Γ if there is no Strong Kleene model in which κ assigns every element of Γ the value 1 and fails to assign ϕ the value 1.

[14] NB: the set complementation is unnecessary in Strong Kleene, since K_3 embraces the Exclusion constraint; however, it is necessary in the dual paraconsistent case, which we briefly sketch below.

Finally, we look at a so-called paraconsistent option, the *Logic of Paradox* or LP. One way of presenting an LP language is in terms of a K_3 language. LP models differ from K_3 models in that they drop the Exclusion constraint, but keep the Exhaustion constraint. The LP valuation scheme ρ is based on the three values $\{1, \frac{1}{2}, 0\}$, and we may leave the clauses for atomic sentences, negation, and disjunction as they were for κ.

The difference appears when we come to consider logical consequence. K_3 was explained in terms of preservation of the value 1 across chains of inference. This is usually put by saying that 1 is the only *designated value* for K_3. For LP, the value $\frac{1}{2}$, in addition to value 1, is designated in the LP scheme ρ. So, Γ implies ϕ iff whenever every element of Γ is designated in a model, so is ϕ. Thus, for ρ, *true in a model* is defined as having either value 1 or $\frac{1}{2}$.

An interpreted formal language is a tool with which issues of truth and issues of logic can be explored, as we have seen with each of our Classical, Strong Kleene, LP examples. We can think of each of these sorts of languages as representing different sorts of logical properties. LP languages, for instance, bring with them a paraconsistent logic, K_3 languages a paracomplete logic, and of course, Classical languages a classical logic. (Again, see §3 for terminology.) There are many other options we could consider, notably relevance logics.[15]

2.2 Background on Truth: Capture and Release

So far, we have explored the idea of an interpreted language, which brings with it a logic. We have looked at options for logic, both classical and non-classical. We now turn our attention to the 'logic' of truth itself.

The term 'logic' here is fraught with difficulty. We are highly ecumenical about logic, and have already surveyed a number of options for what we might think of as *logic proper*. What we now consider is the basic behavior of the truth predicate Tr, described formally, in ways we can incorporate into formal interpreted languages. In some cases, this may require specific features of logic proper, but in many, it is independent of choices of logic. We continue to talk generally about the *logic path* as encompassing both the formal behavior of the truth predicate, and logic proper, as the two are not always easy to separate. But it should be stressed that there are often different issues at stake for the two.

The behavior of the truth predicate — the 'logic' of Tr, if you will — centers around two principles, which have been the focus of attention since seminal work of Tarski [Tar35]. We label these *Capture* and *Release*, which may be represented schematically as follows.

Capture: $\phi \Rightarrow Tr(\ulcorner\phi\urcorner)$.

[15] It is not easy to document the sources of the ideas we have presented in this section. For the machinery of interpreted languages, an extended discussion is found in Cresswell [Cre73], and more recently in Beall and van Fraassen [BvF03]. The Classical language, of course, follows the path set down by Tarski (e.g. [Tar35]). The Strong Kleene language is named after Kleene [Kle52]. The Logic of Paradox was developed by Priest [Pri79], and explored at length in his recent [Pri06a, Pri06b].

Release: $Tr(\ulcorner\phi\urcorner) \Rightarrow \phi$.

We understand '$\Rightarrow$' to be a place-holder for a number of different devices, yielding a number of different principles. (If it is a classical conditional, then these are just the two directions of Tarski's T-schema.) Many approaches to truth, and especially, to the Liar paradox, turn on which such principles are adopted or rejected. Intuitively, all the principles that fall under the schema seek to capture the same idea, that the transitions from $Tr(\ulcorner\phi\urcorner)$ to ϕ and from ϕ to $Tr(\ulcorner\phi\urcorner)$ are basic to truth. They embody something important about what truth is, and flow from our understanding of this predicate. If someone tells you that it is true that kangaroos hop, for instance, you may conclude that according to them, kangaroos hop, without further ado. The leading idea in the study of the formal properties of the truth predicate is that if you understand the right forms of Capture and Release, you understand how the truth predicate works.

We will mention a few important examples of how Capture and Release may be filled in, which will be important in the discussion to come.

Classical conditional (cCC & cCR) This treats '$\Rightarrow$' as the classical material conditional, making Capture and Release two sides of the *Tarski biconditionals* or *T-schema* in (classical) material-conditional form:

$$Tr(\ulcorner\phi\urcorner) \leftrightarrow \phi.$$

Other classical options are available, but we will use this as our main example.[16]

Non-classical conditional (CC & CR) There are various options for non-classical treatments of the conditional. One might stick with the material approach to a conditional, defining it as $\neg\alpha \vee \beta$, but use a non-classical treatment of negation or disjunction to cash out the given 'conditional'. One might, instead, go to a non-classical treatment of a conditional that's not definable in terms of the basic connectives. Prominent options include conditionals of relevance logic and paraconsistent logic, and the more recent work of Field [Fie08a].[17]

Rule form We can replace '$\Rightarrow$' with a rule-based notion. One option is to include a rule of proof, which allows inferences between $Tr(\ulcorner\phi\urcorner)$ and ϕ. We thus have rules:

Rule Capture (RC) $\phi \vdash Tr(\ulcorner\phi\urcorner)$.

Rule Release (RR) $Tr(\ulcorner\phi\urcorner) \vdash \phi$.

[16] For other classical options, see Friedman and Sheard [FS87]. In their terminology, Classical conditional Capture (cCC) is called Tr-In, and Classical conditional Release (cCR) is called Tr-Out.

[17] On relevance (or relevant) and paraconsistent logics see Dunn & Restall [DR02], Restall [Res00], and Priest [Pri02].

Alternatively, we could think of these as sequents in a sequent calculus. Regardless, we will have to work with logics which allow these rules to come out valid.[18]

Fixing on the right form of Capture and Release is one of the important tasks in describing the formal behavior of the truth predicate. Indeed, it is generally taken to be the main task. The reason why is at least clear in the classical setting. cCC and cCR together with facts not having anything to do with truth suffice to fix the extension of the truth predicate. They thus seem to tell us what we want a formal theory of truth to tell us about how truth behaves. The same holds, with some more complications as the logics get more complicated, for non-classical logics.

2.3 Background on the Liar

We have, in passing, mentioned the truth-theoretic paradoxes. We will restrict our attention to a simple form of the Liar paradox. The basic idea of the Liar is well-known: take a sentence that says of itself that it is not true. Then that sentence is true just in case it is not true. Contradiction! We will fill in, slightly, some of the formal details behind this paradox.

To generate the Liar, we assume our language has a truth predicate Tr, and that it has some way of naming sentences and expressing some basic syntax. We will help ourselves to a stock of sentence names of the form $\ulcorner S \urcorner$. (Corner quotes might be understood as Gödel numbers, but for the most part, they may be taken as any appropriate terms naming sentences.)

The Liar, in its simple form, is the result of self-reference (we will not worry if this is essential to the paradox or not). So long as our language is expressive enough, this can be achieved in the usual (Tarskian-Gödelian) ways. With these tools, we can build a canonical Liar sentence: a sentence L which says of itself only that it is not true. In symbols:

$$L := \qquad \neg Tr(\ulcorner L \urcorner)$$

L will be our example of a *Liar sentence*.

Parenthetical remark. If instead of Gödel coding we have names of each sentence, readers can think of our target L as a sentence arising from a name l that denotes the sentence $\neg Tr(l)$. If we use angle brackets for 'structural–descriptive terms', our Liar L arises from a true identity $l = \langle \neg Tr(l) \rangle$. Applying standard identity rules, plus enough classical reasoning (see below), gives the result. *End parenthetical.*

[18] In settings where the deduction theorem holds, the differences between the Rule and Classical conditional forms of Capture and Release tend to be minimal, but in other settings they can be quite important. We also stress that there are other rule forms which are substantially different from RC and RR. Prominent options typically provide closure conditions for theories, telling us if a theory Γ proves ϕ, then Γ proves $Tr(\ulcorner \phi \urcorner)$ as well, and likewise for the Release direction. Rules like these can be very weak. One of the results of Friedman and Sheard [FS87] shows that the collection of all four rules governing Tr and negation is conservative over a weak base theory.

The Liar sentence L leads to a contradiction when combined with Capture and Release in some forms. For instance, the *classical paradox*:

Classical logic + L + cCC + cCR = Contradiction.

The same holds for classical logic and the rule forms of Capture and Release.[19] Of course, once we depart from classical logic, whether or not we have a contradiction, and what the significance of it is, will depend on what conditional or rule is employed, and what the background logic is.

The Liar paradox is thus easy to generate, but does rely on some assumptions, both about the formal behavior of truth, and about logic proper.

3 Nature and Logic

In preceding sections we've discussed the 'nature' and 'logic' paths. We now turn to the crossing. The salient point of crossing, at least for our purposes, comes at the question of the Liar's lesson: what does the Liar teach us about truth? The nature path constrains the logic path by constraining the answers available to the Liar question. We maintain that the nature path does not merely motivate views on the logic path; rather, in some respects, it dictates the available answers to the paradox, and the available views of the logic of truth.

We will show this by asking what the available responses to the Liar are, in light of each of our two representative views of the nature of truth. We will see that they result in very different logical options. Assuming a semantic view of truth, we find that a different account of the formal behavior of the truth predicate is required than we might have expected; but otherwise, the logic may be whatever you will. If classical logic was your starting point, truth according to this view offers no reason to depart from it. In sharp contrast, the transparent view of truth requires the overall logic to be non-classical. We will show this by discussing each view of the nature of truth in turn. We first discuss transparent truth, and then semantic truth.

3.1 Transparent Truth

What does the Liar teach us about truth? In particular, if we embrace the transparent view of truth, what is the lesson of the Liar?

Unlike the case with semantic truth (on which see §3.2 below), the notable lesson is plain: the logic of our language is non-classical if, as per the transparent view, our language enjoys a transparent truth predicate. To see this, consider the following features of classical logic.

ID $\varphi \vdash \varphi$.

LEM $\vdash \varphi \vee \neg\varphi$.

[19] Some more subtlety about just which classical principles lead to inconsistency can be found, again, in Friedman and Sheard [FS87].

EFQ $\varphi, \neg\varphi \vdash \bot$.

RBC If $\varphi \vdash \gamma$ and $\psi \vdash \gamma$ then $\varphi \vee \psi \vdash \gamma$.[20]

Assume, now, that our language has a transparent truth predicate $Tr(x)$, so that $Tr(\ulcorner\alpha\urcorner)$ and α are intersubstitutable in all (non-opaque) contexts, for all sentences α of the language. ID, in turn, gives us RC and RR. Assume, as we have throughout this essay, that our given language is sufficiently rich to generate Liars. Let L be such a Liar, equivalent to $\neg Tr(\ulcorner L\urcorner)$. By RC, we have it that $\neg Tr(\ulcorner L\urcorner)$ implies $Tr(\ulcorner L\urcorner)$. ID gives us that $Tr(\ulcorner L\urcorner)$ implies itself. But, then, $Tr(\ulcorner L\urcorner) \vee \neg Tr(\ulcorner L\urcorner)$, which we have via LEM, implies $Tr(\ulcorner L\urcorner)$, which, via RR implies $\neg Tr(\ulcorner L\urcorner)$. Given EFQ, $\bot$ follows.

The upshot is that no classical transparent truth theory is non-trivial. If our language is classical, then we don't have a non-trivial see-through predicate. On the transparent view, then, the lesson of the Liar is that we do not have classical language. There's no way of getting around this result.

Of course, one might suggest that the transparent truth theorist *restrict* the principles governing 'true' or the like. If one restricts either RR or RC, then the above result is avoided.

What we want to emphasize is that, on the transparent conception, restriction of the principles governing 'true' is simply not an option. After all, at least on the *transparent* view, truth — or 'true' — is a see-through device over the entire language. As such, if the logic enjoys ID, then there's no avoiding RC and RR; the latter follow from ID and intersubstitutability of $Tr(\ulcorner\phi\urcorner)$ and ϕ.

If one suggests that 'true' ought not be transparent over the *whole* language, one needs an argument. Presumably, the argument comes either from the 'nature' of truth or something else. Since the nature route, at least on the transparent conception, is blocked, the argument must be from something else. But what? One *could* point to the issue at hand: viz., Liar-engendered inconsistency. But this is not a reason to restrict RR or RC, at least given a transparent view. What motivates the addition of a transparent device is (practical) expressive difficulty: given our finitude, we want a see-through device over the whole language in order to express generalizations that we could not (in practice) otherwise express. (This is the familiar 'deflationary' story, which we discussed in §1.2.) What the Liar indicates is that our resulting language — the result of adding our see-through predicate — is non-classical, on pain of being otherwise trivial. If one *restricts* RR or RC, one loses the see-through — fully intersubstitutable — feature of transparent truth. In turn, one winds up confronting the same kind of expressive limitations (limitations on generalizations) that one previously had. The natural route, in the end, is not to get rid of transparency in the face of Liars; it is to accept that the given language is non-classical.

So, the lesson of the Liar, *given the transparent conception*, is that our underlying logic is non-classical. The question is: *what* non-classical logic is to underwrite our truth theory? Though rejecting any of LEM, EFQ, RBC (or any

[20] This is sometimes known as $\vee$Elim or, as 'RBC' abbreviates, *reasoning by cases*.

of the steps — even background structural steps) are ‘logical options’, two basic approaches have emerged as the main contenders: paracomplete approaches and paraconsistent approaches. Here, we briefly — briefly — sketch a few of the basic ideas in these different approaches.

3.1.1 Paracomplete

Paracomplete theorists reject that negation is exhaustive; they reject some instances of LEM. The term ‘paracomplete’ means *beyond completeness* — where the relevant ‘complete’ concerns so-called negation-completeness (usually applied to theories).

A paracomplete response to the Liar is one that rejects Liar-instances of LEM. Without the given Liar-instance of LEM, the result in §3.1 is blocked.

A familiar paracomplete theory of transparent truth is Kripke’s [Kri75] Strong Kleene theory (with empty ground model). If you look back at §2.1, wherein we briefly sketch the Strong Kleene scheme, one can see that classical logic is a proper extension of the K_3 logic: anything valid in K_3 is classically valid, but some things are classically valid that aren’t K_3 valid. The important upshot, at least for philosophical purposes, is that a Strong Kleene language, while clearly non-classical, may enjoy a perfectly classical proper part. And this is what comes out in the relevant Kripke picture.

Suppose that our ‘base language’ — the ‘semantic-free’ language to which we add our transparent device — is classical. What Kripke proves is that one may nonetheless enjoy a transparent truth predicate over a language that extends the base language: the base language may be perfectly classical even though, due to Liars in the broader language, our overall — ‘true’-ful — language is non-classical (in fact, paracomplete).

We leave details for other sources, but it is important to note that the relevant Kripke theory is a good example of a (limited) paracomplete theory of transparent truth, one in which much of our language is otherwise entirely classical.

Parenthetical remark. The reason we call Kripke’s theory ‘limited’ is that it fails to have a ‘suitable conditional’, a conditional $\rightarrow$ such that both of the following hold.

cID $\vdash \varphi \rightarrow \varphi$

MPP $\varphi, \varphi \rightarrow \psi \vdash \psi$

While the ‘hook’, namely $\neg\varphi \vee \psi$, satisfies MPP in K_3, it fails to satisfy cID. After all, we don’t have LEM in K_3, and so don’t have $\neg\varphi \vee \varphi$, which is the hook version of cID. The task of extending a K_3 transparent truth theory with a suitable conditional is not easy, due to Curry’s paradox [Bea08a]; however, Field’s recent work [Fie08a] is a major advance. (For related issues and discussion of Field’s paracomplete theory, see Beall [Bea08c].) *End parenthetical.*

3.1.2 Paraconsistent

Another — in fact, dual — approach is paraconsistent, where the logic is Priest's LP.[21] As in §2.1, LP is achieved by keeping all of the Strong Kleene clauses for connectives but designating the 'middle value'. With respect to *truth*, one can dualize the Kripke K_3 'empty-ground' construction: simply stuff all sentences into the intersection of $Tr(x)^+$ and $Tr(x)^-$, and (in effect) run the Kripke march upwards following the LP scheme (which is monotonic in the required way).[22]

Unlike the paracomplete transparent truth theorist, who rejects both the Liar and its negation, the paraconsistent one accepts that the Liar is both true and false — accepting both the Liar and its negation. At least one of us has defended this sort of approach [Bea08d], but we point to it only as one of the two main options for transparent truth.

Parenthetical remark. The noted 'limitation' of the Kripke paracomplete theory similarly plagues the LP theory. In particular, the LP-based transparent truth theory does not have a suitable conditional (in the sense just discussed in §3.1.1). Unlike the K_3 case, we get cID in LP, but we do not get MPP. (A counterexample arises from a sentence α that takes value $\frac{1}{2}$ and a sentence β that takes value 0.) The task of extending an LP transparent truth theory with a suitable conditional is not easy, due (again) to Curry's paradox; however, work by Brady [Bra89], Priest [Pri06b], and Routley–Meyer [RM73] have given some promising options, one of which is advanced and defended in Beall [Bea08d]. *End parenthetical.*

3.2 Semantic Truth and Logic

We have now seen something about how the transparent view of the nature of truth constrains the logic path. In this section, we turn to the semantic view.

For this section, we thus adopt the semantic view of the nature of truth. The result is a more fluid situation than we saw in the case of transparent truth. The transparent view, as we have seen in §3.1, takes the lesson of the Liar to be a non-classical logic for the overall Tr-ful language. In contrast, the semantic view of truth does not start with logical or inferential properties of truth, but rather with the underlying nature of the property of truth. This will allow us to consider what formal principles govern the truth predicate, and how they may function in a paradox-free way. We will find that this may be done without paying much attention to logic proper.

Let us start with a Classical interpreted language as discussed in §2.1, with classical model $\mathcal{M}$ and classical valuation scheme τ. We have already seen in

[21] We should note that Priest's own truth theory is not a transparent truth theory. Indeed, the main point of disagreement between Priest [Pri06b] and Beall [Bea08d] is the relevant truth theory. Both theorists endorse a paraconsistent theory; they differ, in effect, over the behavior of 'true' and the extent of 'true contradictions' — with Beall being much more conservative than Priest.

[22] For constructions along these paraconsistent lines, see Dowden [Dow84], Visser [Vis84], and Woodruff [Woo84], but of particular relevance Brady [Bra89] and Priest [Pri02].

§2.3 that if our language $\mathcal{L}$ contains a truth predicate Tr, and Classical Capture (cCC) and Classical Release (cCR) hold, we have inconsistency. What are we to make of this situation?

The first thing we should note is that the semantic view of truth takes the basic semantic properties of the interpreted languages with which we begin seriously. As we sketched the idea behind the semantic view of truth, it starts with the semantic properties of a language, particularly, those which lead us to assign semantic values to sentences of the language. This is just what our interpreted languages do. Our models show how the values are assigned to the terms and predicates of a language, and the valuation scheme shows how values of sentences are computed from them.

Now, the semantic view of truth takes this apparatus to reveal something metaphysically fundamental about how languages work, and typically, also seeks to explain the metaphysical underpinnings of the formal apparatus of interpreted languages. Our Classical interpreted languages fit very nicely with the rough sketch of the semantic view of truth we offered in §1.1. But regardless of which logic we think is right, it is a metaphysically substantial claim. Most importantly, it is not one that is up for gabs when we come to the Liar and the behavior of the predicate 'true'. Whatever the right semantic properties of a language are, and whatever logic goes with them, is already taken as fixed by the semantic view. For exposition purposes, we will take that logic to be classical logic. If there are reasons to depart from classical logic on the semantic view, they are to be found in the metaphysics of languages, not the formal properties of truth, so this assumption is innocuous for current purposes.[23]

Assuming we were right to opt for classical logic to begin with, the semantic view of truth will not allow us to change it in light of the Liar. This implies that if we are to avoid inconsistency, we must find some way to restrict Capture and Release. This is a hard fact, proved by our classical Liar paradox of §2.3.

Fortunately, in the setting of the semantic view, restricting Capture and Release is a coherent possibility, and indeed, the semantic view provides us with some guidance on how to do so. The semantic view will not completely settle how we may respond to the Liar, and what the formal properties of truth are, but it tells us what determines those properties. This follows, as the semantic view tells us that the function of the truth predicate is to report the semantic values of sentences (in a classical language, those with value 1).

This gives us much, but not quite all, of what we need to know about the formal behavior of the predicate $Tr(x)$. The semantic view indicates we should have $Tr(\ulcorner\phi\urcorner)$ if and only if $|\phi|_{\mathcal{M}} = 1$. As we are assuming that our interpreted language $\mathcal{L}$ already contains Tr, this corresponds to the formal constraint that $|Tr(\ulcorner\phi\urcorner)|_{\mathcal{M}} = 1$ iff $|\phi|_{\mathcal{M}} = 1$. Assuming that the semantics of the language is the fundamental issue, and the truth predicate should accurately report it, this is just what we should want. This is what is often called a *fixed point* property for truth.[24]

[23] The idea that foundational semantic considerations might lead to non-classical logic has been explored by Dummett [Dum59, Dum76, Dum91] and Wright [Wri76, Wri82].

[24] This fixed point property becomes extremely important in the Kripke construction we

The fixed point property, together with classical logic, gives us the force of cCC and cCR. It tells us that $|\phi \rightarrow Tr(\ulcorner\phi\urcorner)|_{\mathcal{M}} = 1$ and $|Tr(\ulcorner\phi\urcorner) \rightarrow \phi|_{\mathcal{M}} = 1$. This makes the Liar a very significant issue for the semantic view of truth, and indeed, more significant than the view often assumes. For, it appears that our philosophical view of truth has already dictated the components of the Liar paradox, including classical logic and Classical Capture and Release (cCC and cCR). Does this show the semantic view to be incoherent?

We believe it does not (at least, one of the authors does). It does not, we will argue, because the semantic view also gives us the resources for a much more nuanced look at the nature of Liar sentences, and what their semantic properties are. In effect, the response to the Liar on the semantic view is a closer examination of L and its semantic properties. However, we will see that along the way, this will show us ways that we can keep to the fixed point property, and still restrict Capture and Release. Thus, we will both reconsider the semantic properties of L, and the underlying behavior of Capture and Release.

We describe three ways to go about this. The first and most familiar, Tarski's hierarchy of languages, will be presented in a way that illustrates the reconsideration of L in the setting of semantic truth. Tarski's hierarchy has been subject to extensive criticism since its inception. Bearing this in mind, we present two further options. The second, the classical restriction strategy, will show how we can reconsider Capture and Release in a semantic setting. Finally, the third, contextualist strategy, shows how both Tarskian and classical restriction ideas can be combined. (One of the authors believes the contextualist strategy is the most promising line of response to the Liar.)

3.2.1 Tarski's Hierarchy

Our first example of a response to the paradox consistent with the semantic view of truth is Tarski's hierarchy of languages and metalanguages. This will illustrate the response of reexamining the Liar sentence L.

As is well-known, Tarski [Tar35] proposes that there is not one truth predicate, but an infinite indexed family of predicates Tr_i.[25] In our framework, Tarski is proposing an infinite hierarchy of interpreted languages. We begin with a language $\mathcal{L}_0$ which does not contain a truth predicate. We then move to a new language $\mathcal{L}_1$ with a truth predicate Tr_1. Tr_1 only applies to sentences with no truth predicate, i.e. sentences of $\mathcal{L}_0$. We extend this to a whole family of languages $\mathcal{L}_{i+1}$, where each $\mathcal{L}_{i+1}$ contains a truth predicate Tr_{i+1} applying only to sentences of $\mathcal{L}_i$. Tr_{i+1} thus applies only to sentences which contain truth predicates among $Tr_o, \ldots, Tr_i$. It does not apply to sentences containing it itself. Each language $\mathcal{L}_{i+1}$ thus functions as a *metalanguage* for $\mathcal{L}_i$, in which the semantic properties of $\mathcal{L}_i$ can be expressed.

Each truth predicate Tr_{i+1} does for the language $\mathcal{L}_i$ exactly what the semantic view of truth asks. We can make sure that Tr_{i+1} is interpreted so as

alluded to in §3.1.1.

[25] Tarski did not then really consider how large this family is. For some more recent work on this issue, see Halbach [Hal97].

to apply to all and only the true sentences of $\mathcal{L}_i$, i.e. all the sentences of $\mathcal{L}_i$ that are assigned value 1 by $\mathcal{M}$ and the classical valuation scheme τ. (We will skip the details, which are familiar from any exposition of Tarski's work. A nice presentation may be found in McGee [McG91].) Along the way, we make true all the instances of Classical Capture and Release. A Tarskian truth predicate does formally just what the semantic view of truth describes philosophically.

What of the Liar? Within the hierarchy of languages, there can be no sentence of any language $\mathcal{L}_i$ which predicates truth of itself. The Liar sentence L of §2.3 simply ceases to exist. It can easily be proved that each language $\mathcal{L}_i$ consistently assigns values to sentences, and so there is no problem of the Liar for languages of the Tarskian hierarchy. Under the Tarskian approach, the Liar genuinely goes away.

As we see it, Tarski's point is a *syntactic* one. He raises the question of just what sort of restrictions may apply to the distribution of truth predicates, and comes to the conclusion that there are strong syntactic restriction. In effect, any well-formed sentence of the Tarskian hierarchy must meet the syntactic restriction of having its truth predicate properly indexed, so as to insure each truth predicate in it applies only to sentences of appropriately lower-level language. Sentences of the Tarskian hierarchy of languages must meet a syntactic requirement of being *well-indexed*.

Thought of this way, Tarski's proposal can be seen as one in the long tradition of care about the nature of *truth bearers*. Though he does not worry about questions of the nature of propositions, Tarski does direct our attention to what sorts of sentences, of what sorts of languages, truth may be appropriately applied. Tarski thus shows us that we can respond to the Liar while keeping the semantic view of truth, along with classical logic, and the basic forms of Capture and Release that go with them. We do so by paying more attention to what sorts of languages enter into the semantic view, and what truth bearers they really provide.

3.2.2 The Classical Restriction Strategy

Tarski's solution to the Liar follows the lead of the semantic view of truth, and does so without finding any reason to depart from classical logic. But it has some well-known costs. Many sentences which seem to us to be intuitively reasonable turn out not even to be well-formed syntactically on Tarski's approach. And, as Kripke [Kri75] famously showed, even when we do not face brute syntactic ill-formedness, the syntactic demand to assign fixed levels to sentences seems to misdescribe the ways we use truth predicates in discourse. Tarski's proposal comes at significant cost in restricting what we can do with truth predicates. It buys a full implementation of a semantic view of truth in a way that avoids paradox, but the cost is very high. Many, if not most, researchers have found this cost too high to pay.

However, we can think of Tarski's theory as an early instance of a more general strategy for resolving the paradox within the semantic view of truth, which we call the *classical restriction* strategy. Tarski's proposal does not restrict

Classical Capture and Release within any given language $\mathcal{L}_i$ of the Tarskian hierarchy. But looked at another way, it does restrict their application. If we widen our view to include sentences outside of the Tarski hierarchy of languages — sentences which contain truth predicates without Tarski's syntactic restrictions — then we can see Tarski's proposal as one to restrict Classical Capture and Release to sentences which meet the syntactic restrictions. As we put it a moment ago in §3.2.1, Tarski can be seen as restricting Capture and Release to sentences which are well-indexed, and so can be placed in the Tarskian hierarchy of languages. Restricting Capture and Release this way does nothing to weaken classical logic.

Tarski's theory can thus be thought of as restricting the domain of application of Classical Capture and Release by *syntactic* means, to retain consistency and keep our model theory classical, and keep the semantic view of truth. Behind this idea we can see a more general theme: we can achieve these goals if we can find principled reasons to restrict Classical Capture and Release. We can find such reasons, the idea goes, by examining the nature of truth bearers. Not every seemingly good sentence really provides us a truth bearer, while Capture and Release need only apply to genuine truth bearers. Tarski does this in rather stringent syntactic terms. The classical restriction strategy pursues this idea more generally, by looking for more plausible, and more flexible, restrictions on genuine truth bearers than Tarski's.

Typically, the classical restriction strategy seeks to reevaluate truth bearers in semantic, rather than syntactic, terms. For instance, it may invoke the idea that there is more to being a truth bearer than simply being a well-formed sentence. Intuitively, we might think about which sentences really express propositions, or otherwise have the right semantic properties. Liar sentences, according to this strategy, are well-formed sentences, but are not semantically in order. Of course, Capture and Release are only to apply to sentences which do have the right semantic properties, and so, will be restricted to avoid the Liar.

A strategy like this needs to be implemented with care. Especially if we are to keep the underlying classical semantics, then we cannot simply deny Liar sentences values in a model. Every sentence in a classical model gets either the value 0 (false) or 1 (true). Every sentence in a classical interpreted language thereby seems to count as semantically 'good'.

At this point, the classical restriction strategy is well-advised to steal a play out of the transparent truth playbook, and in effect, borrow a Kripke fixed point construction from the K_3 paracomplete approach. This gives us a *partial predicate*, which for now, to highlight its intermediate status, we might call Kr. As a fixed point, we have for a K_3 language $|\phi|_{K_3} = |Kr(\ulcorner\phi\urcorner)|_{K_3}$. The Liar sentence L falls in the 'gap' for Kr: $L \notin Kr^+$ and $L \notin Kr^-$. Kr is a fairly good self-applicative truth predicate, and in being a fixed point, it does a fairly good job of reporting the semantic properties of a K_3 language.

We can convert Kr into a classical predicate by the 'closing-off' trick. Assuming Kr is the only non-classical expression of our language, we turn our K_3 model into a classical model by closing off the gap in Kr. We let the extension of Tr be exactly the extension Kr^+, and make the model classical by dropping the antiextension (equivalently, by putting everything in the gap in the antiextension). This is what is often known as the *closed-off Kripke construction*.[26]

Now, with our closed-off Kripke interpretation of Tr, we need to be careful about Capture and Release. The Liar sentence L was in the gap for the original Kripke construction, and so was its negation. Thus, both fall out of the extension of Tr on this interpretation. This will lead to contradiction if Capture and Release apply to them. But, this tells us what the mark of pathological sentences is. Pathological sentences are those which were counted as gappy in the Kripke construction. The negations of these sentences are also pathological. Hence, our pathological sentences and their negations will both fall out of the extension of our interpretation of Tr. We then want Capture and Release to apply only to non-pathological sentences, i.e. those for which we have $Tr(\ulcorner\phi\urcorner)$ or $Tr(\ulcorner\neg\phi\urcorner)$. What we thus need is a restricted combination of Capture and Release, along the lines of a T-schema with an antecedent:

$$[Tr(\ulcorner\phi\urcorner) \vee Tr(\ulcorner\neg\phi\urcorner)] \rightarrow [Tr(\ulcorner\phi\urcorner \leftrightarrow \phi)].$$

This restricts Capture and Release to sentences which are well-behaved on Tr. In fact, if we use the closed-off Kripke construction, we validate this scheme in our Classical interpreted language.

Technically, the closed-off Kripke construction provides us a way to get restricted forms of Classical Capture and Release to come out true in a classical model. It also gives us some idea how to make sense of what 'genuine truth bearers' might be for the classical restriction strategy. Genuine truth bearers are those sentences which are semantically well-behaved. That can be described as those for which we have either $Tr(\ulcorner\phi\urcorner)$ or $Tr(\ulcorner\neg\phi\urcorner)$ come out true in our model. These are sentences whose truth values can be determined in the orderly inductive process by which the Kripke fixed point was built, i.e. those whose semantic value depends on the way the world (or the model) is without too much pathology intervening. It is open to the classical restriction strategy to say that this is a good account of — or at least a good first pass at — what it is to really be a truth bearer. If so, then the restricted forms of Capture and Release do just what they should, as they restrict Capture and Release to genuine truth bearers.

How does this square with the semantic view of truth? The semantic view helps itself to Classical interpreted languages, and sees the job of the truth predicate as simply to report having the semantic value 1 in such a language. In face of the Liar, the classical restriction strategy cannot do this fully. On the closed-off Kripke interpretation, the Liar sentence L falls outside of the extension of Tr, so $\neg Tr(\ulcorner L\urcorner)$ is classically true. Our truth predicate does not reflect this, as it classifies both the Liar and its negation as pathological. Tr does not completely accurately report the classical semantics of our interpreted language.

[26] This was suggested by Kripke [Kri75] himself, and was anticipated by an idea of Parsons [Par74]. The use of Kripke constructions in a classical setting was explored in depth by Feferman [Fef84].

How much of a problem this is may be debated. Those who defend the classical restriction strategy will argue that what we have to do in the face of the Liar is *refine* the semantic view of truth. The idea of the semantic view is that the truth predicate reports the key semantic property of sentences. With the Liar and classical logic and Capture and Release, that property cannot simply be classical semantic value. But, according to the classical restriction strategy, it is a more nuanced property of having a semantic value determined in the right way, as the Kripke construction shows us. The truth predicate on the closed-off Kripke construction does accurately report the semantic values of those sentences. Hence, we might argue, once we refine our notion of what a semantically well-behaved sentence is, i.e. what a genuine truth bearer is, we find that our truth predicate does accurately report the semantic status of genuine truth bearers. Correspondingly, we have Capture and Release for those sentences. Thus, it might be argued, we have done what the semantic view of truth really required.

3.2.3 The Contextualist Strategy

It is not clear whether this defense of the classical restriction strategy really succeeds. The problem with it can be made vivid by the following observation: according to the classical restriction approach, the Liar sentence is not true. It is not assigned the semantic value 1. Furthermore, it is a semantically pathological sentence, i.e. not a genuine truth bearer. For this reason, both it and its negation fall out of the extension of the predicate Tr. But this is just to say that it is not true. It is not true by lights of the classical semantics, and it is also not true by lights of the more nuanced approach to truth bearers the classical restriction strategy proposes. This fact cannot be reported by the classical restriction strategy. It cannot say that the Liar sentence is not true, using the truth predicate Tr. Any attempt to do so winds up with a semantically pathological sentence, rather than a correct report of the semantic status of the Liar. This is the only result possible as any other would drive us back into paradox.

This is what is often called the *Strengthened Liar* paradox, and it is a form of what has come to be called a *revenge* paradox. We will have more to say about revenge paradoxes in §4. For the moment, we may simply note that it raises two problems. First, it suggests we have not really gotten a satisfactory resolution of the Liar, as we still have not accurately explained the Liar's semantic status in a stable way. Perhaps more importantly, it makes clear why one might find the classical restriction strategy unsuccessful as a way of understanding the semantic view of truth. It makes vivid that we have not accurately reported the fundamental semantic status of the Liar, either on a nuanced view, or a crude one. Thus, our truth predicate has yet to live up to the demands of the semantic view.

We (one of us, anyway) thinks the right way out of this problem is to pursue a *contextualist strategy.* Our goal in this paper is not really to advocate for any one approach to the Liar, so we do not argue directly for contextualism, nor do we go into the contextualist strategy in as much depth as such an argument would require. Rather, we present this strategy as a further development of the classical restriction idea. It will thus provide a further, and we think more comprehensive, example of how the Liar may be addressed within the semantic view of truth.

The contextualist strategy, as we understand it, combines features of the classical restriction strategy and the Tarskian one. From the classical restriction strategy, it takes the idea of paying more attention to the semantic properties that make sentences genuine truth bearers. From the Tarskian strategy, it takes the idea of an open-ended hierarchy. Unlike a Tarskian hierarchy, however, it is not strictly a hierarchy of languages syntactically defined.

The contextualist strategy begins with the notion of a truth bearer. It notes that in general, sentences with context-dependent elements cannot be said to be true or false simpliciter, and their behavior is not accurately reflected by the formal apparatus of interpreted languages. What is left out is the role of *context*, which helps determine what such sentences say, and thus what their truth values are.

Relative to a given context, we can think of an interpreted language as representing what speakers can say using their ordinary language in that context. We can thus think of interpreted languages as *indexed* by contexts. The difference between such languages is not in their syntax, but rather in how their sentences are interpreted. We thus can think of a hierarchy of 'languages' indexed by contexts, though it might be better described as a hierarchy of what can be expressed by a language within contexts.

Relative to a fixed context, we can pursue the classical restriction strategy. This will give us an account of a self-applicative (non-Tarskian) truth predicate as used in a language relative to a given context. As we have seen, such a truth predicate might not fully capture the semantic status of every sentence, especially the Liar sentence. It thus may not fully implement the semantic view of truth. But the presence of multiple contexts allows us to work with this fact. Once we have a language with a truth predicate relative to a given context, we can indeed *step back* and observe that according to it, the Liar sentence is not a proper truth bearer.[27] We can then conclude that the Liar sentence is not true. But now, we have at our disposal the resources to see this claim as being made from a *distinct* context. The contextualist proposes that within this sort of reasoning about the semantic status of the Liar sentence is a *context shift*. The conclusion that the Liar sentence is not true is correct, but made from a new 'reflective' context. This can be done without any threat of paradox, as from that new context we can say, with expanded expressive resources, that the Liar sentence as it appeared in the prior context fails to be a proper truth bearer, and so fails to be true. This is a basically Tarskian conclusion. We invoke not a syntactically distinct truth predicate, but rather new expressive resources in a new context, which allow us to draw wider conclusions about truth than we could in the original context.

[27] The term 'stepping back is borrowed from Gauker [Gau06], though Gauker is critical of contextualist proposals of the sort we sketch here.

Many questions may be raised about this sort of proposal: why is there any such context dependence with Liar sentences, why is there a context shift in Liar reasoning, and what is the status of Liar sentences relative to a given context? We will not pursue these here.[28] Rather, we merely comment on how this strategy, however it may be developed, combines Tarskian and classical restriction features. Its Tarskian nature is clear. There is a hierarchy. It is a hierarchy of contexts, and of what can be expressed in those contexts, rather than a hierarchy of languages syntactically individuated. But it is a hierarchy nonetheless. To avoid the Liar, it must have the same open-ended feature as Tarski's own hierarchy.

The contextualist strategy also takes up the classical restriction idea that we can explain the formal behavior of truth by paying attention to what really makes a sentence a well-behaved truth bearer. It does this by paying attention not to static semantic status, as our original classical restriction strategy did, but to how that status may change as context changes. It thus will invoke restricted forms of Capture and Release, along the lines we sketched in §3.2.2, but it will see these restrictions as showing what it takes for a sentence to be a truth bearer in a given context. Strengthened Liar reasoning shows us that this status is apt to change.

Like the Tarskian theory, we believe the contextualist strategy is more faithful to the semantic view of truth. Both views offer truth predicates which correctly report the semantic status of sentences, and both may happily do so using classical logic and semantics. Both do so by placing some limits on what we can say, in some form. The Tarskian view does so rather drastically, ruling much that seemed to be perfectly plausible semantic talk to be syntactic gibberish. The contextualist view, we think, does so rather more gently. It merely says that certain claims can only be made from certain contexts, and you might not be able to say as much as you thought you could without moving to a new context. Because there are such expressive limits, each individual context shows some properties of classical restriction, and we cannot get a truth predicate to behave exactly right on every sentence in any one context. But so long as we allow ourselves to move through contexts judiciously, we can deploy our truth predicate exactly as it was supposed to be deployed to report the semantic status of sentences. It is now the semantic status of sentences *in contexts* that we report, and that is a more delicate matter than our original statement of the semantic view might have envisaged. But it is still the job of truth to report basic semantic status, and the restrictions imposed by the contextualist view, we think, do not undermine this.

[28] They have been pursued at length. The original idea stems from work of Parsons [Par74] and then Burge [Bur79], and has been developed, in very different ways by Barwise and Etchemendy [BE87], Gaifman [Gai92], Glanzberg [Gla01, Gla04a, Gla04b], and Simmons [Sim93]. The term 'reflective context' is from Glanzberg [Gla06a].

3.2.4 The Liar and Semantic Truth

We have now seen three examples of how one might go about addressing the Liar if one starts with a semantic view of the nature truth. This will be enough to draw some conclusions about how the logic and nature paths intersect in this case.

Generally, the semantic view tells us something specific about what the task of resolving the Liar is. If we take the basic standpoint of the semantic view, then our task in the face of paradox is to try to understand better what semantic notions like semantic value are, and how they behave. If we follow the Tarskian line, we do so by paying attention to what sentences are really like. If we follow the contextualist strategy, we do so by paying more attention to what is involved in assigning semantic values in *contexts*. If we follow the classical restriction strategy, we do so by reconsidering what the basic semantic status to be reported by the truth predicate is. Regardless, each strategy pays attention to the fundamental building blocks of semantics, and each find as a result some way to restrict Capture and Release.[29]

The result for each strategy is a more refined picture of the formal properties of Tr — more refined versions of Capture and Release — which allow the truth predicate to function as the semantic view requires and retain consistency. Though at some points, we looked to techniques from non-classical logic, we have seen no independent motivation to make any genuine departures from classical logic. The semantic view of truth tells us nothing directly about logic.[30] Rather, the semantic view constrains us to resolve the paradox by a more careful examination of the behavior of Tr (and related notions) directly, and it provides the means to do so.

4 And Now Revenge

We have now illustrated some ways in which the nature path impacts the logic path. In one case — the transparent view of the nature of truth — the nature path dictates logic proper; particularly, logical options for resolving the Liar paradox. In the other — the semantic view — the nature path requires us to resolve the paradox by restricting the formal principles governing truth (Capture and Release), and it also provides us with resources for doing so. In both cases, we get significant constraints on how we may understand the logic of truth from how we understand the nature of truth, and indeed, we get significantly different constraints depending on the case. The nature path and the logic path thus do indeed meet.

[29] The semantic view also helps, we think, to explain the goal of the *Revision Theory of Truth* of Belnap and Gupta [GB93]. Though, for reasons of space, we will not pursue it in any depth, we can think of this theory as making an even more far-reaching proposal for how to properly understand the fundamental building blocks of semantics. It in effect suggests that we should not think of the basic semantic property as a semantic value at all, but rather as a sequence of such values, under rules of revision.

[30] Or at least, not much. It might tell us that logic must be based on a semantics according to which we can make sense of the semantic view of truth.

We have thus reached the main goal of this paper. As a further application of the point we have made here, we conclude by considering an issue that has proven difficult for the logic path: the problem of so-called 'revenge' paradoxes. The significance of these revenge paradoxes has been a significant question in the literature on the logic of truth. We suggest here that what that significance is, and how revenge paradoxes must be treated, depends on views of the nature of truth. Again, we see the nature path constraining the logic path. In this case, we suggest, seeing how it does helps to answer a question that has preoccupied the logic path itself.

We have already seen something of a 'revenge paradox' in §3.2.3, where we observed that according to the classical restriction account, the Liar sentence is not true. This was presented as a reason to reject the classical restriction strategy. But the style of reasoning it represents applies much more widely, and just what it shows is often hard to assess.

The typical pattern of a revenge objection is as follows. We start with a formal theory, like a theory of truth. We then show that there is some notion X used in the formal construction of the theory which is not expressible in the theory, on pain of Liar-like paradox. Typically, X is *on the surface* closely related to the notion our theory was developed to explain, and is expressible in the target language our theory is supposed to illuminate. Hence, the revenge objection concludes that in not being able to express X, our theory fails to be adequate.

Here is another example. Take as our theory the standard Kripke K_3 construction (empty ground model, least fixed point). The formal object language $\mathcal{L}^+$ of this theory may be the language of arithmetic $\mathcal{L}$ supplemented with Tr. We have seen that Tr serves as a transparent truth predicate for $\mathcal{L}^+$ (though as we observed in §3.1.1, in a limited theory). $\mathcal{L}^+$ is an interpreted language. Its model $\mathcal{M}$ is produced by the Kripkean fixed point technique. As it is a K_3 language, its valuation scheme is the Strong Kleene κ (as we discussed in §2.1). We construct $\mathcal{M}$ in a classical metalanguage suitable for doing some set theory, which we may call $M(\mathcal{L})$. Our metatheory is classical, so we may conclude in it that every sentence of $\mathcal{L}^+$ is true in $\mathcal{M}$ or not, in that either $|\phi|_{\mathcal{M}} = 1$ or it does not. (We may appeal to LEM in our classical metatheory.) Indeed, this notion is *used* in the construction of $\mathcal{M}$.

We now have the set-up for revenge. We have our theory — the interpreted language of the Kripke construction. We also have the metatheoretic notion of being true (having value 1) in $\mathcal{M}$. This is our notion X from above, which, at least on the surface, seems to be related to truth itself. The revenge recipe tells us to consider what would happen if this notion were expressible in our theory. Thus, towards revenge, supposes that there is a predicate $TM(x)$ in $\mathcal{L}^+$ that expresses what we, in $M(\mathcal{L})$, express using 'true in $\mathcal{M}$' (i.e. having value 1). Consider the resulting Liar-like sentence λ equivalent to $\neg TM(\ulcorner\lambda\urcorner)$. A few classical steps, all of which hold in $M(\mathcal{L})$, lead to contradiction. In turn, one concludes that $\mathcal{L}^+$ (interpreted via $\mathcal{M}$) enjoys consistency — more generally, non-triviality — only in virtue of lacking the expressive resources to express TM. Our broader language enjoys this power; indeed, the metalanguage $M(\mathcal{L})$

does. Moreover, TM appears to be a notion of truth — indeed, a notion of truth for $\mathcal{L}^+$, and $\mathcal{L}^+$ cannot express it. In turn, revenge concludes that $\mathcal{L}^+$ is inadequate as a theory of truth: it fails to capture truth for $\mathcal{L}^+$, and so it fails to illuminate how our language, with its expressive resources, can enjoy a consistent (or at least non-trivial) truth predicate.[31]

This is clearly related to the objection raised in §3.2.3, which also argued for the inadequacy of a theory based on its failure to capture a model-theoretic notion used in the construction of the theory. There are a great many related forms of revenge paradox, and it is not our aim to explain their structure in detail. Rather, we wish to consider the question of how effective such revenge paradoxes are as objections to a theory, and note that the answer depends on what view of the nature of truth is assumed.[32]

As we did in §3, we will show this by considering how the question should be answered under the transparent and semantic views of the nature of truth, in turn. We begin with the transparent view. As we have said, this view holds that 'true' is only a logical, express device; it is brought in for practical reasons, to overcome practical limitations of finite time and space. As the transparent view is a species of deflationism, it importantly does not see 'true' as naming any important property — a fortiori, not some property essentially tied to semantics or meaning. On 'deflationist' views in general, fundamental semantic properties — including anything that determines linguistic meaning — are not to be understood along *truth-conditional* lines at all.[33] (Using a transparent truth predicate, we may state truth conditions, but this can play no explanatory role in semantics.) Something similar holds for the notion of *truth in a model*. This cannot be seen by deflationist views as a formal representation of a fundamental semantic property, not even an idealized one. The transparent view of truth does not — cannot — hold that truth in a model is a property of fundamental importance to understanding the nature of languages. But, then, the models involved in giving a model theory for a formal theory of truth are at best convenient tools, which do not themselves amount to anything of theoretical importance. They are basically heuristic guides to logic.[34]

With this in mind, we can see that the transparent view of truth allows revenge paradoxes to be dismissed in many cases. It is crucial to the revenge problem that failing to express the revenge notion X amounts to a failure of adequacy. In our example cases, it is argued by the revenge objection that failure to express a model-theoretic notion used in constructing a theory amounts to a failure of adequacy. But given the transparent view of truth, even if such an inexpressibility result holds, it is not obviously a defect. If a model-theoretic

[31] For a more leisurely and more detailed discussion of revenge, see the papers in [Bea08c].

[32] Our discussion is brief here. For more details, see [Bea08b], Field [Fie08b], and Shapiro [Sha08].

[33] Frequently deflationists opt for something like conceptual role semantics. See Field [Fie86, Fie94] for discussion. Strictly speaking, the transparent view does not need to take a stand on semantics, as it is a view about the nature of the truth predicate.

[34] Model theory can also be an important technical device, which provides a host of techniques of interest to logicians. See Dummett [Dum78, Dum91] and Field [Fie08a] and Dummett for discussion of these issues.

notion, like truth in a model, is merely a heuristic device for specifying a logic, we cannot conclude that failure to express it is any kind of failure of adequacy for a theory meant to capture the notion of truth. The most we can conclude is that there is some classical model-theoretic notion (e.g., truth in a model) that is used in the metalanguage of the construction but that our formal theory cannot non-trivially express.[35] Though this could turn out to be a problem for the theory, and show it to be inadequate, we have yet to see any reason why. More importantly, simply displaying the 'revenge' form of the paradox does not demonstrate that it is. If we hold a transparent view of the nature of truth, the model-theoretic revenge paradoxes in logic fail to show the inadequacy of a theory of truth. To the transparent view, revenge — at least of the 'model-theoretic' sort we have discussed — is not serious.

What of the semantic view? We have already seen that the semantic view leads to very different conclusions about revenge, such as the one we drew in §3.2.3. We have also seen that the semantic view takes a very different stance towards issues of semantics and model theory. The version of the semantic view we sketched in §1.1 starts with the idea that there is a fundamental semantic property described by assigning (the right sort of) semantic values to sentences. It also holds, as we discussed in §3.2, that the notion of truth in a model provides a theoretically useful way of representing this notion formally. According to the semantic view of the nature of truth, the model-theoretic notion of truth in a model reveals an important property, of explanatory significance. The job of the term 'true' is to report that property. Thus, where the transparent view sees truth and truth conditions as merely heuristic notions, the semantic view sees them as fundamental explanatory notions for semantic theory and for meaning in general. Indeed, the core of the theory, on this view, is the theory of truth conditions, semantic values, and related notions.[36]

If we think of truth this way, then the revenge charge of inadequacy is very plausible, and we believe, in some cases, devastating.[37] In particular, the model-theoretic revenge problem, of the sort we just reviewed, is serious for the semantic view of truth, as it was in §3.2.3. If the notion of truth in a model provides a fundamental semantic concept, and indeed, the very one that our target notion of truth is supposed to capture, then failing to express it is just the failure of our theory to do its intended job. If we take the semantic view of truth, then typical revenge paradoxes, including the two we have seen here, are

[35] Whether this is the case really does depend on the details. For useful discussion, see Field [Fie08b] and Shapiro [Sha08].

[36] We have taken a model-theoretic stance towards theories of truth conditions, but that is but one tool that might be invoked here. So long as there is a substantial semantic theory, which yields truth conditions in some form, one can hold the semantic view of truth. Hence, as we mentioned in §1.1, taking a more Davidsonian view of semantics is no impediment to the semantic view. The idea that natural languages may be modeled with the tools of formal logic, in either model-theoretic or proof-theoretic terms, is basic to much of contemporary semantic theory. It is common today to grant that these tools apply differently in the semantics of natural language than they do in logic proper, though from time to time, the stronger view has been advocated (cf. Montague [Mon70]).

[37] One of us (viz., Glanzberg), who prefers the contextualist approach to the Liar, sees the right form of 'revenge' as simply *the* issue of the Liar.

just ways to observe that a theory fails to live up to its own goals. That — in purest form — is inadequacy.

Thus, what the significance of revenge paradoxes is turns out to be strongly influenced by view of the nature of truth. This reinforces our main conclusion, that the two paths — nature and logic — do meet, and do so in interesting ways. On the surface and in practice, the two have seemed not to intersect. We hope to have illustrated that this is not so. We have shown that responses to the Liar are strongly influenced by the nature path. Whether such a response results in a non-classical logic, or restrictions on the principles governing truth, depends on view of the nature of truth. Whether or not revenge paradoxes are significant does as well. Not only do the two paths meet, but the logic path can learn something from following the nature path.

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