Can u do that?*

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In his 'On t and u and what they can do' [9], Greg Restall presents an apparent problem for a handful of well-known non-classical solutions to paradoxes like the liar. In this paper, we argue that there is a problem only if classical logic – or classical-enough logic – is presupposed.

1 Background

Many have thought that invoking non-classical logic – in particular, a paracomplete (gappy) or paraconsistent (glutty) logic – is the correct response to the liar and related paradoxes. At the most basic level, the target non-classical idea is that some expressions, like 'all and only the true propositions', do not behave as we would expect from classical logic. Non-classical theorists argue that the class of all and only the truths is either incomplete or inconsistent: when you truly speak of *all and only truths* (or, dually, untruths), you're either leaving some truths out, or you're letting some *un*truths in. Truth, in a slogan, is either gappy or glutty.

Non-classicality is not a glib or easy-way-out response to the paradoxes. Innocuous-seeming notions can turn out to be philosophically substantial. Moreover, apparently correct forms of reasoning can turn out to be incorrect. To take a (non-arbitrarily selected) example, a glut theorist must hold that the following argument form is not in general valid.

Either p is true or q is true. But p is untrue. So q is true.

This argument form may strike our ears as acceptable. But if p is a truth value glut, then the inference fails to preserve truth. According to glut theorists, the inference breaks down

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in inconsistent contexts: if the subject matter involves gluts, then the inference is to be rejected. And the liar paradox, according to such theorists, shows that *truth* is exactly the kind of subject matter that yields inconsistency.

In the dual case, concerning gap theorists, it is inferences relying on excluded middle that are rejected in incomplete contexts. We focus mainly on the glut side, but much of what we say is available, in dual form, to corresponding gap theorists.

2 Restall's argument

In arguing against non-classical solutions, it does no good to use inferences that are unacceptable to the given non-classical theorists. Instead, one needs to show that *uncontentious* forms of reasoning raise trouble. And that is what Restall's argument [9] against target non-classical theories aims to do: show that, via only acceptable forms of reasoning, the target theories are strapped with absurdity.

The argument, which turns on two seemingly banal propositions, goes as follows. Take the proposition t to be the conjunction all truths, and u the disjunction of all untruths, respectively. Formally, t is characterized by the rule that t entails p iff p is true, and u is characterized by the rule that p entails u iff p is untrue. Let us also have – as Restall's target, non-classical theories do have – some conditional $\rightarrow$ which obeys modus ponens, identity, antecedent strengthening and consequent weakening. Define

$$p \Rightarrow q := (p \wedge t) \rightarrow (q \vee u)$$

and a corresponding negation

$$-p := p \Rightarrow u$$

Then $-p$ is true if and only if p is untrue. So, apparently, t and u allow us to reintroduce so-called boolean negation. This is cause for concern for target non-classical theorists; the liar paradox expressed with boolean negation is trivializing – leading directly to absurdity [4, 8]. Similarly, the conditional $p \Rightarrow q$, defined as above, seems very much like the material conditional (true iff either p is untrue or q is true), and so $\Rightarrow$ seems to allow Curry's paradox [6], and therefore triviality again.[1]

[1]In languages in which the material conditional is the only conditional (or, at least, the conditional underwriting the sentences at issue), Curry's paradox and the liar are hardly distinct; however, Restall's target theorists [1, 3, 5, 7] have conditionals, represented here as $\rightarrow$, that are brought in precisely to overcome the limitations of the material conditional.

Restall's argument, then, is that the (target) non-classical solutions to paradoxes fall to absurdity if they recognize such seemingly banal statements such as the conjunction of all truths or the disjunction or all untruths: by theory-sanctioned inferences, such notions drive the target theories to triviality. But is that right? A closer look at u is required.

3 Reply: looking at u

Little t is the conjunction of all and only truths; little u is the disjunction of all and only untruths. As non-classical theorists, we have already questioned the meaning of 'all and only'. Accordingly, we now question the reasoning leading to apparently disastrous conclusions.

Towards generating curry-paradoxical trouble, Restall argues that

$$p, p \Rightarrow q \therefore q$$

is truth-preserving. Suppose $p \Rightarrow q$ is true, and that p is true. Then $q \vee u$ follows from the truth of t and *modus ponens*. Does the truth of q now follow? No! At least on the glutty account, u has a true disjunct (just as t has a false conjunct), since – on that account – some truths are among the untruths. Hence, inasmuch as u is the disjunction of all untruths, u itself is true (in addition to being false or untrue). So u cannot be discarded in favor of q; the inference would not necessarily be truth preserving.[2]

By comparison, the glut theorist can, and typically does, recognize an 'entirely untrue' (so to speak) statement $\perp$, which is an 'explosive' (triviality-implying) statement: $\perp$ implies everything. So the argument from $q \vee \perp$ to q is valid. But u is not $\perp$; u is untrue, while $\perp$ is *absurd*. (In particular, it is not the case that u entails $\perp$.) And distinguishing untruth from absurdity is the *sine qua non* of glutty theories of truth. So $\Rightarrow$, defined as above, is truth-preserving only if we characterize u in a consistent and complete way – that is, only if we equate inconsistency with absurdity and so forgo the essence of the target non-classical ideas. Restall is correct that, if either p is untrue or q is true, then $p \Rightarrow q$ is true. But when $p \Rightarrow q$ is true, it does not necessarily follow that either p is untrue or q is true. As Restall says (for a quite different reason), the defined $\Rightarrow$ is not the material conditional. For reasons just given, no devastating material-like conditional has been recovered from

[2] From a glut-theoretic perspective, the occurrence of u in Restall's multiple-conclusion deduction cannot legitimately be used to close the branch.

intensional operators – contrary to Restall's argument.

The reply with respect to liar-like paradox is the same. A liar-like sentence g, expressed with u-negation, would look like this:

$$g \leftrightarrow (g \Rightarrow u)$$

The target glutty perspective delivers u-excluded middle and, in particular, its g-instance: $g \vee -g$. This, in turn, delivers the u-contradiction $g \wedge -g$, which in turn implies both g and $g \Rightarrow u$, and so implies g and $(g \wedge t) \rightarrow (u \vee u)$. We then have the following valid derivation.

$$g$$

$$g \wedge t$$

$$(g \wedge t) \rightarrow (u \vee u)$$

$$\therefore u$$

This is valid because t is true (and adjunction valid), $u \vee u$ is equivalent to u, and $\rightarrow$ obeys modus ponens. Hence, given a u-liar sentence, the glut theorist is committed to u. But this is not a problem: u is true; it is a large disjunction and, on the glutty account, some of its disjuncts are true. A liar-like contradiction using a different notion like $p \rightarrow \perp$ really would be trouble—but this merely re-illustrates that untruth and absurdity are distinct.[3] So appeals to u can't recover boolean negation, unless we presuppose boolean negation in reasoning about u.

We have focused on replies from a glut perspective, but a similar (in fact, dual) argument is available on behalf of gap theorists. Gap theorists may reject u-excluded middle, reject that $p \vee (p \Rightarrow u)$ holds for all p. In a thoroughgoing paracomplete theory [5], talk of 'all truths' comes up short of boolean expectations. The only (sort of) argument we can see for u-excluded middle invokes something too close to excluded middle itself. In particular, suppose that p is true, in which case, by addition, $p \vee (p \Rightarrow u)$ is true. No problem. Suppose, on the other hand, that p is untrue, in which case, by definition of u, we get that $p \Rightarrow u$ is true. Again, $p \vee (p \Rightarrow u)$ follows. Hence, from 'reasoning by cases',

[3] About a $\perp$-liar: we don't get either of g or $g \rightarrow \perp$ from $g \leftrightarrow (g \Rightarrow \perp)$. The failure of contraction blocks the fastest path to $\perp$; and there is no reason to think that $\perp$-excluded middle holds – that is, that $p \vee (p \rightarrow \perp)$ is true. Moreover, as shown in [1: 121], at least some of the target glut theories that add the so-called Ackermann constant t (for present purposes, not obviously different from Restall's little t) are non-trivial, and these are theories that contain an absurdity constant $\perp$.

which holds in target paracomplete theories, *that either p is true or p is untrue* implies u-excluded middle: $p \vee (p \Rightarrow u)$. But why would a paracomplete theorist find this argument for u-excluded middle any less objectionable than we glut theorists find Restall's argument (discussed above) for the alleged validity of $\Rightarrow$-*modus ponens*?

4 Closing remarks

When we attempt to encode pre-theoretic concepts into a formal logical system, we often meet with unexpected consequences. A situation directly analogous to that with t and u arises in the standard (non-normal-worlds) semantics for target non-classical theories – semantics that critically invoke the distinction between 'normal worlds' and 'non-normal worlds'. In short (but skipping details here), when we consider a proposition n true at all and only *normal points* (or, if you want, a proposition $\hat{n}$ true at all and only non-normal points), we seem to get a Restall-like troubling conditional: a conditional $p \mapsto q$ defined as $(p \wedge n) \rightarrow q$ (or, if you want, $p \rightarrow (q \vee \hat{n})$) seems, at least on the surface, to yield curry-paradoxical absurdity [2] or triviality-inducing liar paradox.[4] But the reply to such an apparent problem is the same: that 'all and only normal points' turns out to have non-classical import. Statement n is not classically behaved – or, at least, does not behave as one would expect from a classical view. And the same goes, as we have argued, for Restall's t and u. These propositions cannot be construed as picking out exactly – all and only – the boolean-negation-delineated truths and falsehoods; there simply is no such thing by target non-classical lights.

A sceptic might complain that a non-classical solution to the paradoxes has cost us our ability to use or understand words like 'untrue'. This worry turns out to have less to it than appears on first sight. ('I am embarrassed to admit that I once tentatively voiced this worry in print.' Field [1: 362, fn].) The glut theorist, for example, *can* use and understand notions such as 'all and only truths' or the like as expressed in the expected locutions. In fact, we (authors) do fully agree that there is a class of all and only untrue propositions, and say so now: it's defined by little u. Dually, the true propositions are defined by t. But that very claim – that there is a class of all and only untrue propositions – is itself subject

[4]For discussion of target (non-normal-worlds) semantics, see Priest 1987, Brady 2006, Beall 2009, or Field 2008. Details are different, but the structure of the semantics is the same. As Beall [2] shows, the conditional $\mapsto$, so defined, seems to be what Restall calls a 'contracting conditional' [10], one that is at the very heart of the curry paradox. If that's right, then such non-classical theories immediately confront absurdity.

to non-classicality.

In the end, Restall's real charge is that 'all and only' fails to behave in a complete and consistent fashion; and we fully agree with him, as would the corresponding gap theorists. But we non-classical theorists would never have claimed complete-and-consistent behavior on behalf of 'all and only', or a fortiori on behalf of *all and only untruths*. In a slogan, our reply to Restall is just this: u is what u is – even if u isn't what you expected.[5]

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