Defending Logical Pluralism

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1 Introducing Logical Pluralism

We are pluralists about logical consequence [1]. We hold that there is more than one sense in which arguments may be *deductively valid*, that these senses are equally good, and equally deserving of the name *deductive validity*.

Our pluralism starts with our analysis of consequence. This analysis of consequence is not idiosyncratic. We agree with Richard Jeffrey, and with many other philosophers of logic about how logical consequence is to be defined. To quote Jeffrey:

> Formal logic is the science of deduction. It aims to provide systematic means for telling whether or not given conclusions follow from given premises, i.e., whether arguments are valid or invalid . . .
>
> Validity is easily defined:
>
> > A *valid* argument is one whose conclusion is true in every case in which all its premises are true.
>
> Then the mark of validity is absence of counterexamples, cases in which all premises are true but the conclusion is false.
>
> Difficulties in applying this definition arise from difficulties in canvassing the cases mentioned in it . . . [6, page 1]

We agree that deductive validity is a matter of the preservation of truth in all cases. An argument is valid when there is no counterexample to it: that is, there is no case in which the premises are true and in which the conclusion is not true. We call this account of validity (V) for short.

Our *pluralism* stems from the *difficulties* to which Jeffrey alludes. We hold that there is more than one notion of *case* which may be substituted (correctly) into the defining scheme for validity. In particular, we think that there are at least *three* kinds of case which may be used to give at least three different *logics*.[1]

If you take cases to be *worlds* (or world-like models) then the scheme results in a variety of *classical logic*.[2] According to this logic, from contradictions, anything at all follows, for contradictions are true in no possible world. According to this logic, the law of excluded middle holds, for worlds are complete: for every proposition, either it or its negation is true.

[1] In fact, we hold that there are many more than these three, but the three at hand are useful in illustrating our view.

[2] The subtleties of whether this ought to be first-order or second-order logic, of whether modal operators are logical constants, and all of the other issues raised by the use of *models* will not detain us here.

However, the study of logic in the twentieth century has shown us that worlds (or world-like models) are not the only 'cases' which may be used to evaluate propositions. We take it that two other sorts of cases have equal rights to be used in the scheme defining validity.

There is an important tradition within logic which takes *constructive* reasoning seriously. According to this tradition, deductive validity respects the limits of 'construction.' An inference is valid just when any construction of the premises is also a construction of the conclusion. Constructions obey familiar laws. A construction of a conjunction is a construction of both conjuncts. A construction of a disjunction is a construction of one of the disjuncts. The novelty comes with the treatment of negation. Constructions may be *incomplete*. A construction may not decide between a proposition and its negation. However, we have a construction of a negation $\sim A$ just when there is no *further* construction (extending this one) which constructs the negand A. That is, we have 'verified' $\sim A$ when no further verification can possibly verify A.

On this tradition the valid argument forms differ from those of classical logic. A simple case is the failure of $A \vee \sim A$. Not all constructions will verify the law of the excluded middle. Similarly, a construction may verify $\sim\sim A$ (no further construction will verify $\sim A$) without itself being a construction of A.

For pluralists like us, constructive reasoning provides a *different* logic, which is equally applicable in the analysis of reasoning as is classical logic. It differs from classical logic by providing a different account of the kinds of 'cases' in play in the definition of validity.

We do not think that plurality stops with constructive and classical reasoning. We think that further insights in logic in the twentieth century are fruitfully understood in a similar way. Contemporary work in *relevant* logic and in *situated reasoning* gives us another account with another kind of 'case.' Situations, like constructions, may be incomplete. However, situations may also be *inconsistent*. For a *relevant* logic, the inference from a contradiction $A \wedge \sim A$ to an arbitrary conclusion B may well fail, because there is an *impossible* situation in which $A \wedge \sim A$ is true, but in which B fails. These situations are ways that the world (or that part of the world) cannot be. Situations are related by *compatibility*. Two situations are compatible if nothing in the one conflicts with what is the case in the other. It follows that some situations — the inconsistent ones — are not even compatible with themselves.[3] Given a relation of compatibility, it makes sense to say that a negation $\sim A$ is true in a situation s just when A is not true in any *compatible* situation t. For if t is compatible with s, and $\sim A$ is true in s, then we cannot have A true in t, lest t be incompatible with s. Conversely, if A is not true in *any* situation compatible with s, it would seem that s has truly ruled the truth of A out, so $\sim A$ is true in s.

We take it that each of these logical systems (classical, constructive, and relevant) are well motivated, and that each has its own sense of 'case' which is suitable for substitution into the definition of logical validity. There are different logical systems equally deserving of the name *logic*.

[3] However, they may be compatible with other situations. For if s is inconsistent about something, and t carries absolutely no information about that, then there may well be nothing in s which conflicts with t. For more on this interpretation of situations, see Restall's "Negation in Relevant Logics" [14].

This was the view presented in our previous paper [1]. In this paper we will defend and clarify our view. We will not be presenting new arguments for it. That task was attempted in our original paper, and will be resumed in other papers in the future. However, we take it that the way in which pluralism is able to respond to its critics is useful material to be used in weighing up its attractiveness or otherwise as a philosophy of logic.

In the remainder of this paper we will deal with a number of arguments presented against our version of logical pluralism. The end result will be a revised and clarified statement of logical pluralism.

2 The Peircean Objection, and Logic's Point

As above, pluralism says that there are many equally true instances of (V). To say that an instance of (V) is *true* is to say, among other things, that the given instance specifies a consequence relation (a logic) over our language — each of these specifying senses in which some claims *(deductively) follow from* others.[4] Most philosophers seem to recognise that 'follows from' is ambiguous between an inductive sense and a deductive sense. Pluralists say that the ambiguity goes further: On the deductive side of the inductive–deductive divide there is another, many-branching divide, each branch of this being specified by one among the many true instances of (V).

Now, among the many true instances of (V) are instances in which (V)'s cases are filled with inconsistent or incomplete situations — that is, the given models involve inconsistent or incomplete points (or both). Suppose, with virtually all philosophers, that the only way our (actual) world could've been is a complete and consistent one. If this is so, then the inconsistent or incomplete situations involved in some instances of (V) are simply *impossible cases* or *impossible worlds* — they are, as it were, "ways the world *can't* be."[5]

This raises the first and perhaps most natural objection to pluralism, *the Peircean objection*. This objection stems from a particular view of Logic, which is nicely illustrated in some remarks by C. S. Peirce.[6]

First, consider the following two arguments each of which is taken from one of Peirce's [9, page 274] discussions of Aristotle's logic. Assume, as did Peirce, that the conclusion of the first and the premise of the second are necessarily true and necessarily false, respectively.

> Chameleons assume the color of objects upon which they rest, therefore everything is what it is.
>
> Some parts are greater than their wholes, therefore the eating of green fruit proves invariably fatal.

[4]One may substitute for 'claims' a term that reflects one's favoured theory of truth-bearers — 'sentences', 'propositions', perhaps even 'thoughts', etc. Unless context demands otherwise we use all of these terms interchangeably. Our pluralism is intended to be neutral on the issue of *ultimate truth bearers*.

[5]For discussion of the 'ways' analogy, see Lewis [7] and, for present purposes, Restall's "Ways things can't be" [13].

[6]What we call the *Peircean objection* is an objection that comes up frequently in discussions of our pluralism — or, at least, the objection *has* come up frequently when we've presented pluralism to various audiences. We are especially grateful to Greg O'Keefe and Colin Cheyne for pressing the point on different occasions. We do not claim to be historically accurate with respect to Peirce's considered views.

Of these two arguments Peirce says the following:[7]

> [A]lthough I think Aristotle or any other man of good sense would take this view [*viz.*, that, "as [Aristotle] would have said, this is no reasoning at all"], I propose to reject it, and to consider both the above reasonings as [valid]. *My reason is, that such things are of no practical importance whatever — for as long as reasoning does not lead us astray, the whole purpose of logic is fulfilled* ... of syllogism. In this I am by no means alone. Even in ancient times, many logicans took the same ground ... [9, page 275, italics ours]

According to this view *the whole point of Logic* is to ensure that our arguments do not "lead us astray." And the idea seems to be that if there's no *possible* situation in which one's premises are true but one's conclusion fails to be true, then one can never be *led astray*. Consequently, *impossible* cases — cases in which, say, $A \wedge \sim A$ is true or $A \vee \sim A$ fails to be true — are simply beside the point of Logic. It may be that, as Peirce puts it, the arguments strike one as "no reasoning at all." It may be that one recognises a genuine sense (or more) in which 'The eating of green fruit proves invariably fatal' *doesn't follow from* 'Some parts are greater than their wholes'. Still, according to the going objection, such arguments are to be counted among the valid provided that there is no *possible* situation in which the premises are true but the conclusion fails to be true. If this condition is met, the whole point of Logic is satisfied. So goes the Peircean objection.

Our pluralism agrees with some points in this objection but disagrees with others. Before detailing these points it may be useful to look at the objection in a slightly different guise.

The Peircean objection can be further illustrated by focusing on familiar divisions among paraconsistent logicians. A *paraconsistent logic* is any logic in which explosion — $A \wedge \sim A \vdash B$ — fails. One natural way of developing a paraconsistent logic involves the use of inconsistent situations — situations in which $A \wedge \sim A$ is true.

Within this tradition there are at least three important brands of paraconsistent logician: Regular Dialethists, Light Dialethists, and Non-Dialethic Paraconsistentists. Each of these logicians, as above, admits inconsistent situations into her models — she uses inconsistent situations in her approach to filling out (V), as we might say. However, the three types of logician differ with respect to their views on the status of such inconsistent situations. For dialethists (both regular and the low fat 'light' variety) these inconsistencies are genuine *possibilities*. Things *could be* inconsistent. The regular dialethist goes further by holding that the actual world is inconsistent. A light dialethist rejects this conclusion.[8] So, dialethists maintain that their models represent genuine possibilities.

[7]Note that Peirce uses 'sound' where we have inserted 'valid'. There is no doubt that Peirce meant to be speaking of what contemporary logicians call 'validity', for Peirce agrees that the premise of the second argument is false.

[8]Perhaps more accurately, the light dialethist doesn't *knowingly* hold that the actual world is inconsistent. Some cases of light dialethism collapse into strong dialethism, depending on the logic involved. See Restall [12] for discussion. That paper introduced the typology of regular and light dialethism.

Against this, the non-dialethic paraconsistentist disagrees. She admits inconsistent situations, but she sees the situations (points) in her models as being *impossible*; they are but ways the world *cannot* be. In other words, a non-dialethic paraconsistentist says that among (V)'s cases there are inconsistent ones; however, she differs from her dialethic colleagues in holding that such inconsistent cases are *impossible*.

Now, the non-dialethic paraconsistentist provides a nice example of the sort of position against which the Peircean objection responds. According to the objection the only sorts of cases with which Logic is concerned are *possible* cases — true-blue ways things *could have* been. There is no point at all in formulating cases that aren't really possible. Accordingly, there is no point in being a non-dialethic paraconsistentist, at least as far as Logic is concerned.

What can we say? Clearly, the driving force behind the objection is the idea that "the whole purpose of logic" affords but one logic — one and only one true instance of (V). But what *is* the purpose of logic, according to the going objection? The heart of the answer, as Peirce puts it, is that we never be "led astray". But what is this?

Being *led astray* is relative to some fixed direction. With respect to reasoning such "directions" may be such things as *reasoning constructively*, or perhaps *reasoning relevantly*, or *et cetera*. But, then, suppose one's "direction" is that of *relevance* (as it were). In this case, what would it be to be "led astray?" The answer seems to be straightforward: One would be led astray if one's conclusion didn't conform to the canons of relevance. Better put: One would be led astray if one's conclusion failed to *follow relevantly* from one's premises. Likewise, of course, for the constructive path — or the constructive "direction."

So, even if the whole purpose of Logic is to avoid being led astray, there seems to be more than one logic that may arise given this purpose. One must stay on the right path, to be sure, but there's certainly more than one path along which one might trek.[9] For this reason, the Peircean objection seems to lose force.

The objection loses even more force when one considers the question: Is it really the case that the whole purpose of Logic is to avoid being led astray? Whatever the metaphor of *being led astray* utlimately amounts to, one thing seems to be correct: Logic is concerned with specifying *logical consequence* — specifying what follows from what. But, then, given this as a goal, the Peircean objection quickly deflates.

Consider the case with the non-dialethic paraconsistentist. By Peircean lights (as here characterised), once a logician ceases to see her models as representing genuine *possibilities*, she is required, in the name of Logic (and, perhaps, simplicity), to nix those models — or, at any rate, nix the *impossible* situations involved. But this is simply bad policy. If the goal of the weak paraconsistentist is to model logical consequence, then she may well require impossible situations to produce the best models.

What is interesting in Peirce's own remarks is that he seems to admit to there being an obvious sense in which (say) explosion is "no reasoning at all."

[9]Alas, this makes Logic sound alarmingly puritanical or alarmingly sporty. The analogies, of course, should be taken with a grain of salt.

Put differently: Peirce seems to admit to a sense of 'follows from' in which an arbitrary B simply *does not follow from* an arbitrary $A \wedge \sim A$. By pluralist lights, what Peirce was recognising here was a genuine consequence relation. And according to pluralism, it is the job of Logic to record this relation by filling in (V)'s cases appropriately — where propriety, here, may well call for *impossible* cases.[10]

Of course, as in the previous section, we agree with the Peircean who maintains that validity is truth-preservation over all *possible* cases. That's not at issue. What's at issue is whether that's *all* there is to validity. Against the Peircean we say 'no'; some consequence relations require quantification over other kinds of cases.

3 Reasoning Between Logics

Another common objection is closely related to the Peircean one. The objection arises in various guises, but perhaps the simplest version runs thus.

According to pluralism there are many different logics each of which specifies a genuine consequence relation. But now there's a problem. Which of these many logics governs your reasoning about how many logics there really are? In other words, which logic ought to govern your reasoning about the nature of logic itself? And indeed, which logic ought to govern your reasoning concerning *that* question — the question of which logic ought to govern your reasoning about the nature of logic itself?

The objection, in short, is that pluralism has no non-arbitrary way of choosing which among its many (alleged) logics ought to govern our reasoning in the debate between monism vs pluralism. This, in turn, is supposed to be a mark against pluralism, given that monism, for obvious reasons, has no trouble at all in specifying which among its logics *it* chooses.

What can we say? There are many things to say, but we will focus on the most important point. The most important point is that though Logic and reasoning are intimately connected, we reject the view that all reasoning — or, perhaps, all *rational* reasoning, or etc. — corresponds to some logic. In particular, then, the apparent dilemma posed by the current objection may be merely that: apparent. Provided that some reasoning can be done that is, in some sense, independent of any logic, the objection does not seem to get off the ground.

The question is: In what sense is logic independent of reasoning? Before answering this question let us make clear an important sense in which logic is tied to reasoning. The answer is simple: Logic provides models (or systematic accounts) of *logical consequence*. By doing this Logic provides models with which to analyse and/or evaluate reasoning. Every logic, then, will model reasoning; but not all reasoning need be modeled by some logic.

Our view on this matter is summarised well be Gilbert Harman:

[10] Note that J. Michael Dunn, who is a non-dialethic paraconsistentist, gives the following short but sufficient reason for filling in (V) with inconsistent cases: "I believe that there is a sense of 'entails' (or 'implies') in which it simply is not true that a contradiction entails (or implies) any old sentence whatsoever" [2, page 9].

> If there is a connection between standard principles of logic and principles of reasoning, it is not immediately obvious. There is a gap. We can't just state principles of logic and suppose that we have said something precise about reasoning. [4, page 6]

Once this point is recognised, the force of the current objection seems to diminish. We should note, however, that some, including Harman himself, will take the point too far. With respect to the (alleged) implications of the distinction between reasoning and logic Harman claims the following:

> [C]onsider a defense of a "logic of entailment," which observes (1) in standard logic a contradiction logically implies any proposition at all, and (2) one is not justified in responding to the discovery that one's view is inconsistent by inferring anything whatsoever, concluding that (3) a new logic is needed (Meyer [8]). This line of thought loses plausibility if rules of inference or reasoning are distinguished from rules of implication or argument. [4, page 6]

We think that such a defense of a new logic is perfectly appropriate. After all, it seems that if (2) above is true, then there is very likely to be a sense in which arbitrary B does *not follow from* arbitrary $A \wedge {\sim} A$. If this is right, then we maintain that (3) is correct — a new logic is needed indeed, or *would be* needed if there weren't one already.

On the other hand, if (1)–(3) in any way presuppose that all reasoning corresponds to a logic, or that logical monism — the view that (V) has but one true instance — is correct, then we too would reject (1)–(3), or at any rate reject the given presuppositions.

With respect to the objection concerning "reasoning about logics", then, our response amounts to this: We resist its core presupposition that all reasoning correpsonds to some logic. While this point can be taken too far we think that it's an important point to keep in mind, especially when reasoning about the nature of Logic. Of course, it *may* be that, as the going objection presupposes, reasoning about Logic does indeed correspond to or "require" some given logic; however, until a strong case is made for this, we set the objection aside.

4 Truth Conditions and Meaning Variance

The foregoing objections focus broadly on the idea that there is more than one correct logic. The next objection is more specifically targeted at details of our pluralism.

The objection at hand stems from the connection between meaning and truth conditions. According to our pluralism the truth conditions of the one and the same connective can be given in different ways. But this means we have to respond to an important problem. Priest puts the problem this way:

> [Beall and Restall argue that] we can give the truth conditions for the connectives in different ways. Thus, we may give either intuitionist truth conditions or classical truth conditions. If we do the former, the result is a notion of validity that is constructive, that is,

> tighter than classical validity, but which it is perfectly legitimate to use for certain ends . . .
>
> We can indeed give different truth conditions. But the results are not equally legitimate. The two give us, in effect, different theories of vernacular connectives: they cannot both be right.

Priest's point may be illustrated with an example. For us, the following three clauses are *each* explications of the behaviour of negation:

- *Classical*: $\sim A$ is true in a world w if and only if A is not true in w.
- *Constructive*: $\sim A$ is true in a construction c if and only if there is no construction d extending c in which A is true.
- *Relevant*: $\sim A$ is true in a situation s if and only if for each situation t compatible with s, A is not true in t.

These are, without doubt, *different* accounts of conditions under which a negation is true. For Priest, it follows that we have different theories of the one connective, and that they cannot all be correct theories. At most one can be a true account of the behaviour of negation, and *that* one will presumably feature in the One True Logic.

If this is the case, it is a fatal objection to our theory. We propose different logics as equally good accounts of what follows from what. If the defining features of each logic is incompatible with those of each other logic, then the only way in which each logic may be equally good is for each to be equally *false*. This is unpalatable.

So, we must respond to this objection. How can each clause for negation be *equally* good? The response is reasonably straightforward. How can any number of different claims about a thing be equally true claims about that thing? They can be equally true simply in virtue of being *incomplete* claims about the object in view. If JC says that Graham Priest is a philosopher, if Greg says that he is a Marxist, and if someone else says that he is accomplished at karate, then it does not follow that we are not all correct. None of us, however, has told the *complete* story of Graham Priest. Each of us has described one of his features. The descriptions can be (and are) jointly true.

We think that the same holds of our different accounts of negation. They do not *conflict*, they are *incomplete*. The classical clause gives an account of when a negation is true in a world. The constructive clause gives an account of when a negation is true in a construction. The relevant clause does the same for situations. Each clause picks out a different feature of negation.[11]

However, it is one thing to *claim* that three statements are consistent. It is another thing entirely to justify the claim. And consistency claims do sometimes need justifying. It may not be implausible that there is a Marxist philosopher accomplished at karate, but it may be less plausible that there is a Marxist philosopher who doubles as a bank manager by day and a used car salesman by night. One might seriously doubt whether *those* properties are consistent. How could one show that they *are* consistent? The most straightforward way is to find a

[11] Or one could just as truly say that each clause picks out a feature of the *objects* described, be they worlds, constructions or situations.

candidate: to exhibit a Marxist philosopher banker and used car salesman. But if there is none such person, or if they are difficult to find, you could at least tell a *story* which makes it plausible that the required mix of interests, character traits and opportunities can coexist to produce such a bizarre specimen of humanity. Such a story may well help explain the compossibility of character traits, and hence, justify the claim that they are compatible.[12]

Can we do the same sort of thing for our views on negation? We cannot tell a *biographical* story explaining how these three clauses come to be true together, and any account of what the worlds, constructions and situations actually are would no doubt be very contentious. We can, however, provide a *model* of how these things could be. Models play a important explanatory role in many disciplines because they provide a picture of how different theoretical claims could be true. This is what we will provide, to show how our three different claims about the behaviour of negation can be true together.

In any model in which there worlds, constructions and situations, the first issue to deal with is the relationship between them. In this model, we will take worlds to be a kind of construction — worlds decide every proposition to be true or false (and not both) so the job they do is done just as well by *final* constructions: those constructions which are not extended by any other constructions. So, our model will contain a family of constructions, some of which are *worlds*. These constructions are ordered by the partial order of *extension*, which we represent by '$\sqsubseteq$'. Worlds are endpoints in this ordering: if w is a world, there is no $c \neq w$ where $w \sqsubseteq c$.

It seems plausible to suppose that there are *many* worlds — in particular, that every construction is extended by a world. This makes sense, since all constructions are consistent, and any consistent information is possible, and hence it is true in some world.

More can be said about the relationship between constructions and worlds, but we need not spend time with this here. It is enough to note that *however* the story is elaborated, negation works as expected, in worlds as well as constructions. If in *all* constructions we use the standard clause for negation: $\sim A$ is true in c iff A is not true in d whenever $c \sqsubseteq d$, then worlds behave classically. If w is a world, then $\sim A$ holds in w if and only if for each construction $c \sqsupseteq w$, A is not true in c. But w is the only construction satisfying this condition, so the condition is simply that A is not true in w. Choosing worlds to be final constructions gets the condition *exactly* right.[13]

The relationship between worlds and situations is similar. Situations come ordered, just like constructions. As with constructions, if $s \sqsubseteq t$, and A holds in s, we take A to hold in t too. However, they are related also by a symmetric relation of *compatibility*.[14] Compatibility (denoted 'C') is related to ordering in

[12]We think, though, that the epistemology of compatibility judgements is a difficult and important area of epistemology. Platitudes about it get you only so far. For a sketch of the difficulty of going further, consider van Inwagen's recent paper on modal epistemology [5].

[13]This modelling is not intended to drive home any philosophical point about the 'nature' of worlds or of constructions. If you take constructions and worlds to be completely different *sorts* of things, the point still stands. The things true at worlds can be exactly those things true at *final* constructions.

[14]We assume that compatibility is symmetric here for ease of presentation. There are accounts in which compatibility fails to be symmetric [3, 14] but admittedly, it is hard to see why this might be the case in the application in question here.

the following way: If $s \sqsubseteq s'$ and $t \sqsubseteq t'$ then $s'Ct'$, then sCt too. If there is no conflict between s' and t', then there is no conflict between any situation inside s' with any situation inside t' either. The relation C is used to model negation: $\sim A$ is true in s if and only if whenever sCt, A is not true in t. A negation is true in a situation if and only if the negand is not true in every compatible situation.

We do not assume that compatibility is reflexive. There may be situations which are incompatible with themselves. These situations are inconsistent. They are 'impossible'. Not in the sense that they do not exist (one may well be a realist about these impossible situations) but in the sense that they can never be *actualised*. They are never part of any possible world. This suggests a relationship between situations and worlds. A world is a *consistent*, *complete situation*. A situation s is consistent if and only if sCs.[15] A situation s is *complete* if and only if whenever sCt, $t \sqsubseteq s$.[16] A world can be modelled as a consistent complete situation, for in these situations, $\sim A$ is true if and only if A is not true.

Now, we seem to have an embarrasment of riches. Worlds are a kind of construction, and worlds are a kind of situation. To finalise the story we need to relate constructions and situations. If worlds are both constructions and situations, then at least some constructions are situations. We need to decide whether we can relate constructions and situations more intimately than that. We will try that. Let's take *all* constructions to be situations. This means that we have some situations (namely, our constructions) in which $\sim\sim A$ holds, but A fails.[17] Clearly not all situations are constructions. In some inconsistent situations we have $A \wedge \sim A$, and no construction can be like that. Other situations are so incomplete as to not verify each instance of $\sim(A \wedge \sim A)$.

How can we distinguish our constructions from our other situations? Constructions are special. They are not only consistent, but *explicitly* so. In each construction, we have $\sim(A \wedge \sim A)$, and therefore, if c is a construction, and cCs, we must have sCs. If a construction is compatible with some situation, that situation must be consistent. Constructions may well be seriously incomplete, but they still carry the information that the world is not inconsistent.

To complete our picture, we need to verify that the clause for negation in constructions works hand in hand with the clause for negation in situations. To do this we need one small further assumption, extending our assumption of the plenitude of worlds. Now, we will assume that if s and t are consistent situations, which are compatible, then there is a world w of which s and t are both a part. So if sCs, tCt, and sCt, then there is a world w where $s, t \sqsubseteq w$. This is a plausible assumption to make. If s and t are consistent and are compatible, then they can be true together — they are part of some world

Once we make this assumption, then we can show that if c is a construction, the constructive clause for negation agrees with the situation-theoretic clause. If A fails in every situation compatible with c, it follows that it fails in any constructing extending c, since any such construction is compatible with itself, and hence, is compatible with c. Conversely, if A fails in every construction extend-

[15]You may verify that if sCs, then A and $\sim A$ cannot both be true in s.

[16]You may verify that if s is complete, then at least one of A and $\sim A$ is true in s.

[17]That means that the notion of relevant consequence elaborated here is not the standard notion of relevant consequence, for which $\sim\sim A$ entails A. The loss does not seem too great. To maintain the validity of $\sim\sim A \vdash A$, we must pry apart situations and constructions. The only situation which is a construction is a world, for in all constructions, we have $\sim\sim(A \vee \sim A)$, and therefore, if the construction is a situation, we have $A \vee \sim A$, and hence, either A or $\sim A$, for each A.

ing c, then if s is a situation compatible with A, it follows that s is consistent (since constructions are compatible with consistent situations only, because of their verification of $\sim(A \wedge \sim A)$), and by the plenitude of worlds assumption, since cCs and c and s are consistent, there is some world extending both. But worlds are constructions, and we know that A fails in that world. So A must fail in s too. So if A fails in every construction extending c, it must fail in every situation compatible with c too.

So, this model provides a way for each clause to be true. Our analyses of negation may be true together. They are not inconsistent.

This means that each clause gives some information about the behaviour of negation, and none, or at most one, gives the *complete* story of which negations are true in which states. This may raise a raise a further objection to our pluralist view:

> If a logic gives a merely partial account of negation, then it does not give the *whole* story of negation. Only a logic which does *that* (and which gives the whole story of other connectives) is rightly called 'Logic'.

For this argument to have any bite, the notion of the *whole story* of a connective must be elaborated. What could this mean? It certainly does not mean that a logic should determine the truth value of every sentence involving the world 'not'. That would make logic a truly all-embracing theory, determining the truth of every claim. A more plausible claim is that the whole story required is the analysis of which negations are true in what states. In our model given above, in which worlds are (complete) constructions which are themselves (consistent) situations, it is the relevant story which gives the all-encompassing story of negation. And therefore, by this reasoning, it is this relevant logic alone which deserves the name 'Logic'.

We respond to this interpretation of the argument by resisting the premise. A logic need not determine the truth of each sentence in every state, since the states in question may not be relevant to the distinctions drawn by the logic itself. We think that since classical logic is logic, it is no skin off the nose of classical logic that it does not determine which sentences are true in which *constructions*, or that it can not find any difference between $\sim\sim A$ and A. Classical consequence is still an important notion of consequence, as deserving of the name 'consequence' as any other. We will argue this point a little more in later sections. Before that, we need spend some time on the consequences of our pluralist account for theories of *meaning*.

Given that these different clauses for connectives are true together, we have consequences for our theories of truth conditions, and perhaps also of meaning. For each of our clauses for negation gives a different account of the conditions under which a negation is true. The classical clause tells of when a negation is true in a world, the constructive clause, when a negation is true in a construction, and the relevant clause, when a negation is true in a situation. Each could plausibly be described as giving *truth conditions* for negations. Now many have thought that the meaning of a sentence or an utterance is intimately connected with its truth conditions — perhaps its meaning is determined by its truth conditions, or is even constituted by those truth conditions. However you come down on this issue, a pluralist view of logic has consequences for your view of truth conditions, and perhaps therefore, of meaning.

For what are the conditions under which a negation is true? We can *all* answer

- $\sim A$ is true if and only if A is not true.

for truth apt claims A. This is not *too* controversial.[18] Call this condition the *trivial* condition for negation. However, the trivial condition is not the end of the story of the conditions under which a negation is true. It is one thing to think that a negation is true if and only if its negand is true. It is another to hold to *any* of our three clauses governing negation. For it does not even follow from the trivial condition that $\sim A$ is true in a world w if and only if A is not true in that world. For the trivial condition might be true, but not *necessarily* true. For biconditionals may be true without being necessary: that is, they may be true without being true in all worlds. The classical clause for negation strengthens the trivial clause by extending it to *all* worlds. Similarly, the constructive and relevant clauses, far from being inconsistent with the trivial clause, *extend* the trivial clause to deal with constructions and situations.

What does this mean for theories of meaning which take truth conditions seriously? At the very least we need to take greater care. There are many different notions of truth conditions, at least one for each different kind of case in which claims may be evaluated as true or false.

5 The Purpose of Logic: Reasoning about Situations

Priest's criticisms of pluralism does not end with truth conditions. Another line of response focusses on the *purpose* of logical consequence. For Priest, the aim of logic is defined as follows:

> When we reason, we reason about some situation, state of affairs, or other. The situation may be actual or hypothetical. We reason to establish what holds in that situation given what we know, or assume, about that situation. I will call this truth-preservation (forward), though it is not actually truth that is in question unless the situation we are reasoning about is itself actual. The *point* of deduction, then, is to give us a set of canons that are guaranteed to preserve truth in this sense. A valid inference is therefore one such that in all the situations in which the premises hold, the conclusion holds [10].

There is much that we agree with here. For Priest, and for us, logical consequence is about the preservation of truth in all situations. The pluralist, however, disagrees with Priest in holding that the domain of quantification of this 'all' is not set once for ever. It can vary, and its variance gives rise to different logics. For Priest, the domain does not vary. The quantifier is *universal*. We must admit that this is a *natural* assumption to make. After all, the quantifier is a universal one. (The onus is on us to show that the domain of this universal

[18] Intuitionists and dialetheists may well disagree with this, but to do so, they will have to take issue with either the T-scheme or the contraposability of or the transitivity of the biconditional (and this is sometimes explicitly done [11, 15]). For $\sim A$ is true if and only if $\sim A$; and since A if and only if A is true, we have $\sim A$ is true if and only if A is not true.

quantifier shifts. We have attempted in our previous paper to show that this is the case.)

This view of deduction gives Priest a ready response to our pluralism. We will first consider what Priest has to say about our espousal of constructive logic, given that we have *also* admitted a stronger logic (in our case, classical logic) as logical consequence.

> ... it is indeed the case that one could decide to operate with just the constructive part of the correct logic (assuming that that logic is not itself constructive) if there were a *point* to doing so. A point might be provided by the fact that the conclusions obtained in this way contain more information than conclusions proved more generally. It may therefore be a useful *instrumental* technique. I noted that a pure logic may be applied for many purposes, and instrumental purposes are purposes. Hence this is a case of applying a logic for a different end, and we have already seen that different applications may require different logics. Note, though, that it does not follow from this that conclusions obtained using non-constructive principles are themselves defective in any way. The things so proved are guaranteed, in fact, to hold in the situation about which we are reasoning, simply by the definition of validity. [10]

For Priest, if there is a *logic*, properly so called, which is not constructive, then constructive logic can have at most instrumental value. It is just as much 'logic' as the restriction of deduction to propositional logic rather than predicate logic. It says nothing about the nature of valid deduction, for as Priest says, the conclusion of a valid but constructively invalid argument still holds in all of the cases in which the premises are true.

Our response, as pluralists, to this objection falls into two parts. First, we must confess a little difficulty with the distinction between *instrumental* ends and the 'proper' end of logical consequence. There is no doubt that there is a distinction to be drawn in this vicinity, but it seems very difficult to establish the nature of the distinction in question. For *all* of our ends in deduction seem instrumental. We are interested in classical validity because we wish to avoid stepping from truth to falsehood. Classically valid arguments provide a way to ensure that you do not. For any classically valid argument you *necessarily* will not step from truth to falsity. This seems just as much an instrumental end as the desire to avoid conclusions which are too long, or which contain three negation signs in a row.

Yet, there is a difference between those ends which have something to do with the *content* of our claims and their *consequences* on the one hand, and those which are foreign to the concerns of deductive logic. Unfortunately, we have little to say to make this distinction precise, except to point to our definition of deductive validity. What makes classical, constructive, and relevant logics *logic* is their analysis in terms of the cases in which claims are true. They depend not on extraneous features of the representations of claims (in the way that mere 'syntactic filters' do).[19]

[19]We have in mind here those accounts of relevant consequence according to which an argument from A to B is valid if and only if the argument is classically valid, and in addition A and B share a propositional atom. This notion of 'consequence' is not transitive, and so, it does not fall under our account (V) of validity. We do not see this as flaw in (V).

Our second response is to reject Priest's conclusion that "it does follow that conclusions obtained by non-constructive principles are themselves defective in any way." This is precisely what we wish to reject. These conclusions are defective by virtue of being *non-constructive*. And by our lights, this means that there *are* cases in which the premises are true and in which the conclusion is not true. These are cases which reflect the considerations of constructive reasoning: they are *constructions*.

However, it may appear then that the shoe is now on the other foot. If constructions *are* situations, then does it not follow that *logic* must be constructive? Is not any non-constructive reasoning *bad*? For any non-constructive argument, there is a construction in which the premises hold and the conclusion does not. Answering this objection requires getting to the heart of the nature of reasoning, and the relationship between reasoning and the situations we reason about. For Priest, if there are *constructive* situations, logic must itself be constructive.

> . . . it is only truth preservation over *all* situations that is, strictly speaking, validity. One of the points of deductive logic is that it will work, come what may: we do not have to worry about anything except the premises . . . this is not to say that in practice one may not reason as if one were using a different, stronger, notion of validity, one appropriate to a more limited class of situations. But this is not because one has changed logical allegiances: it is simply because one is allowed to invoke contingent properties of the domain in question. [10]

This is an important disagreement. Clarifying the difference between our pluralism and Priest's position will help us clarify our position in an important way.

Our disagreement with Priest is the step from the truism that logic works, come what may (with which we agree) to the conclusion that only truth preservation over *all* situations counts as validity. Before explaining how we think this step fails, we will first explain some senses in which it succeeds.

One sense in which the step succeeds is when we reason about the physically possible. In doing this, we restrict our attention to worlds in which the actual physical laws are respected. Physical consequence is consequence in all physically possible worlds. There is a clear sense in which this consequence is not logic, because it does not work 'come what may'. To get from premises to the conclusion you use also the extra premises of the physical laws. The class of cases in play is *contingently* restricted. The laws could have been other than they are.

There is a second case in which the Priest's conclusion seems appropriate. If we agree with Priest that there are true contradictions, yet we wish to restrict our reasoning to particular phenomena which are known to be consistent. The reasoning about this consistent phenomenon may proceed classically with its assumptions of consistency and completeness. Yet again, the admissibility of classical inference is only contingent. Now it is not extra facts that are admitted, but it is the *domain* about which we are reasoning which is restricted. Again, the cases in play do not exhaust the class of possibilities, and the logic in play does not work, 'come what may'.

With these two examples it may well seem that Priest's point is well taken. However, we still resist it because of an important disanalogy with our pluralism. For the strongest logic we countenance is classical logic. And for us, classical logic is the logic of *all* possible worlds. Therefore, in a very important sense, classical logic works, come what may. No matter what happens, if you have a classically valid argument, if the premises are true, the conclusion *must* be true too. Classical logic works, come what may.

However, it does not follow that in every *case* in which the premises are true, so is the conclusion. For example, $A \vee \sim A$ is classically valid, yet it does not hold in all *situations*. There are incomplete situations in which $A \vee \sim A$ fails. There are incomplete constructions. This does not mean that $A \vee \sim A$ could fail to be true. For we admit that $A \vee \sim A$ is necessarily true. It is true, come what may. However, it is not true in all situations (not all situations decide on the matter of A or $\sim A$), and nor is it true in all constructions.

In no sense is the restriction to possible worlds a restriction invoking "contingent properties of the domain in question." Whenever you have an inference from premises to conclusion which is classically valid, you will never, and *can* never step from truth to untruth. This is the sense in which classical logic is logic, universal and always applicable. We agree with Priest's premise that mere contingent or domain restrictions are not appropriate in a logic. Logic applies, come what may. For us, *each* logic applies, come what may.

We hold, of course that classical logic is *not* universally applicable in the sense of dictating what is true in each and every case. Not all situations and constructions are closed under classical consequence. But that does not mean that classical validity is not *validity*. For it is still true that if the premises of a classically valid argument are true, the conclusion *must* be true too. Of course, classical consequence does not do *every* job required of deductive validity. Other logics are better suited to some of these tasks. That is why we need a plurality of logics.

6 One Final Argument

There is one remaining argument in Priest's paper which needs a response.

> We often reason about some sitation or other; call it s; suppose that s is in different classes of situations, say, K_1 and K_2. Should one use the notion of validity appropriate for K_1 or for K_2? we cannot give the answer 'both' here. Take some inference that is valid in K_1 but not K_2, $\alpha \vdash \beta$, and suppose that we know (or assume) α; are we, or are we not entitled to accept β? Either we are or we are not. [Footnote: It could, I suppose, be maintained that there is no fact of this matter; that both answers to the question are equally correct. But this is relativism (about truth, and so about validity). And B&R (section 6) maintain that their pluralism is not a relativism.] A natural reply is that we should use the notion of validity appropriate to the smallest class of situations that s is in; in this case, presumably $K_1 \cap K_2$. But if we should, indeed, apply the notion of validity appropriate to the smallest class that s is in, then we should apply the notion appropriate to $\{s\}$. Thus, the valid inferences are those that have a premise false in s, or whose conclusion is true in s. In other words, it is now pluralism that has become vacuous. [10]

Priest's argument proposes to reduce our pluralism to a vacuous position. Take the situation as Priest describes, in which α is true in s, and the inference from α to β is accorded as valid with respect to K_1 (without loss of generality, let K_1 be the class of all worlds, and the resulting logic, classical validity) but not with respect to K_2 (take this to be the class of all situations, and the resulting logic a kind of relevant validity). If α is true in s, and if s is a member of K_1, then by the K_1 validity of the inference from α to β, it follows that β is true in s. That is not at issue. The *pluralism* in our position comes from the plural senses of *entitlement*, not any plurality with respect to *truth*.

We spell out this plurality with respect to entitlement as follows: the inference from α to β is not K_2 valid, and as a result, there is some situation in K_2 in which α is true and in which β fails. For concreteness' sake, let us suppose that β is $\gamma \vee {\sim}\gamma$ for some γ irrelevant to α. Then indeed there is a situation in which α is true but in which $\gamma \vee {\sim}\gamma$ fails.

The inference from α to $\gamma \vee {\sim}\gamma$ is *valid* in the usual classical sense: if α is true, then of necessity $\gamma \vee {\sim}\gamma$ is true. There is no possibility (that is, no possible world) in which α is true and in which $\gamma \vee {\sim}\gamma$ fails. So, we are *classically entitled* to infer $\gamma \vee {\sim}\gamma$ from α.

But of course, this inference is not as 'good' as others in which inference is really inference *from* its premises. In this inference, α has done nothing to contribute to β. If our canons of entitlement are strict enough to include a condition of relevance, then this inference fails the test. This failure is recorded in our semantics by the existence of a situation in which α holds but in which $\gamma \vee {\sim}\gamma$ fails. This situation is not our original s, of course, and it is not in the class K_1 of worlds. But nonetheless, it contributes to the relevant invalidity of the argument we are considering as applied to s. The argument is invalid, and so, we are *not relevantly entitled* to infer $\gamma \vee {\sim}\gamma$ from α.

Note that this is not a relativism (or even a pluralism) about truth. We can agree that 'truth in a case' is completely determinate and non-relative. A final analogy might help illustrate the point. Consider a graph of a function $f : \mathbb{R} \to \mathbb{R}$, which is, say, continuous but not differentiable. We can ask if this graph is smooth or not. Surely there is a fact of the matter about its smoothness. Either it is smooth or it is not. But the notion of smoothness can be formalised in different ways, equally acceptably. If you take smoothness to be 'no jumps' then continuity might be an acceptable formalisation. On that account of smoothness, f is smooth. If on the other hand, you take smoothness to be 'no bumps' then continuity is not so acceptable, but differentiability might be more so. On this account, f is not smooth. This 'pluralism' about smoothness is akin to our pluralism about consequence.[20] Pluralism about smoothness does not involve some kind of pluralism about the locations of points on graphs — the graph of f is totally determinate given the function. Similarly, pluralism about the validity of an argument need involve no pluralism or relativism about the truth or otherwise of the premises in different cases. The source of the pluralism is the pretheoretic notion of validity, which can be made precise in different ways.

[20] Of course, this analogy breaks down at a point. We hold that validity is usefully characterised by an account (V) which 'wraps up' the different ways of making the notion precise. It is not clear that *smoothness* can be wrapped up in the same way.

7 Closing Remarks

Graham Priest poses the question: "Logic: One or Many?" Our answer is "both".

> *One*: There is precisely one core notion of logical consequence, and that notion is captured in schema (V).
>
> *Many*: There are many true instances of (V), each of which specifies a different consequence relation governing our language.

This one-many answer is what we call 'pluralism'. While we think that there is much to say in support of pluralism, this paper has aimed only at defending and clarifying it. Our hope is that this aim has been reached.[21]

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