Free of detachment: logic, rationality, and gluts*

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1 Introduction

Glut theorists maintain that some sentences are true and false – true with true negations. The best examples of such gluts are antinomies (e.g., the liar paradox). My aim here is not to argue for glut theory; that has been done elsewhere [3, 5, 6, 21, 22, 35, 36, 46]. My aim is to address a longstanding and very common objection to glut theories. The objection, roughly, is:

1. Glut theory involves the invalidity of material modus ponens (or, on its contemporary tag, *material detachment*): $\{A, A \supset B\} \therefore B$ is invalid.

2. Material detachment is central to our theoretical work.

3. Without some way to understand how we pursue common theoretical work without the validity of detachment, glut theory remains not only a highly radical idea; it is simply implausible – full stop.

The dominant reply by glut theorists over the last four decades [4, 5, 6, 17, 31, 38, 36, 51] is to concede (1), for reasons briefly reviewed in §5 below, but, towards answering (2) and (3), proceed on a quest for (non-material) detachable conditionals, generally following the lead of Anderson and Belnap's work and that of others in the 'relevance' tradition

*Forthcoming *Noûs*. Not final copy.

[1, 2, 17, 23, 31, 32, 38, 39, 42, 47, 50].[1] But as Dunn, Meyer and Routley noted early on [33], Curry's paradox riddles the quest for detachable conditionals with severe problems; and overcoming the problems makes for very complicated, philosophically awkward semantics [9], and indeed often engenders the need to find yet other detachable conditionals to serve other pressing needs (e.g., restricted quantification) [10, 12, 19]. The quest for detachable conditionals that are suitable for glut theories can – and often does – appear to the informed observer as the wrong direction of reply to the problem of material non-detachment.[2]

My aim, in this paper, is not to discuss the standard response to the non-detachment objection (viz., the quest for a non-material detachable conditional), but rather to show that the objection can be met at a much earlier stage. Nothing I say, in this work, argues against the quest for detachable conditionals. What I hope to show is that the given objection can be met via an alternative route, one that doesn't point beyond the material conditional to a (highly complicated) detachable conditional.

For reasons given in §5 below, I grant (1); and, as above, I concentrate entirely on the material conditional. I focus on answering the challenge in (3), and, in doing so, address (2). My aim is not so much the coming up of brand-new ideas as it is the aim of putting good ideas together to answer the given challenge. Towards that end, I review, in very broad strokes, what I take to be a plausible and relatively uncontroversial picture of logic and rational inquiry (e.g., rational 'change in view'), due largely to Harman [28, 29]. In turn, I rehearse, in very broad strokes, two traditional roles of logic in rational theory change (i.e., rationally changing what we accept and reject). Putting these two pieces together – Harman's basic lessons and the traditional roles of logic in rational inquiry – affords a plausible route towards understanding our freedom from (the validity of) detachment.

[1] Note well: though many pioneers of relevance logic (certainly, Anderson and Belnap) were not themselves glut theorists (accepting theories in which a sentence and its negation are true), their work has been applied towards target glut theories in the 'quest' for suitable conditionals. (And this quest has gone beyond glut theorists, most recently Field [26] follows a similar quest on behalf of non-glutty but nonetheless non-classical theories.)

[2] For a brief and recent recap of broad issues, see Beall [6, ch. 2]; and for some of the complexity involved in the 'suitable conditionals' see Field's recent work [26].

2 Harman on logic and reasoning

Gilbert Harman [29] argued for what is now, at least in broad terms, a largely uncontroversial but fundamental thesis: namely, that *logic* and *theory of reasoning* (or theory of rational change in view, etc.) are distinct. For present purposes, the important distinction is as follows.[3]

1. Logic is the theory of *implication* or *consequence*: it is about *what logically follows from what.* Given a theory (a set of sentences), *logic* tells you what follows, logically, from that theory.

2. The theory of reasoning is the theory of rational change in view: it is about *what, rationally, are available 'view changes'.* Given a theory, the theory of reasoning tells you the rationally available options: for example, whether it's (rationally) open to accept the given theory and reject another theory, whether it's (rationally) closed to expand the given sort of theory by including such-n-so sort of theory, and more.

What is important to note is that *logic* concerns a relation over sentences or propositions or etc.; the relation is (logical) consequence, validity, implication, or entailment.[4] The relations at the heart of a *theory of reasoning* are much more complicated: they concern agents, acceptance, rejection, etc. Logic says nothing at all about what one ought to accept or reject; theories of reasoning or of rational change in view focus on such matters, leaving logic to tell us about validity, consequence, etc.

Other features that distinguish logic from reasoning, according to Harman (and I shall follow suit), is that the former is 'monotonic' while the latter, let me say, is 'defeasible' (enjoying a sort of non-monotonic structure). In particular, *adding* to a valid argument won't remove its validity; that's the monotonicity of logic.[5] Rational change in view (changing what one accepts, rejects, etc.), on the other hand, exhibits a more take-back or 'non-monotonic' pattern: it may be rational to accept A given your initial

[3] I should note that Harman argues for a variety of points, some very detailed, some broad. For my purposes, it is only the very broad distinctions and general direction(s) of Harman's ideas that are relevant here. Indeed, I disagree with Harman on some points of detail; but I leave these for other debates.

[4] For purposes of discussion, I treat these terms (consequence, etc.) as synonyms.

[5] Valuable work is done on so-called *non-monotonic logics* [30], but on the current way of talking, such 'logics' are only so called. My aim is not to quibble over terminology, but rather to straighten (or, if nothing else, stipulate) terminology for purposes of this paper.

theory X, but irrational to accept A given your expanded theory $X \cup Y$. It may be, for example, that the combination of theory Y and sentence A results in (rationally) unacceptable incoherence.

According to Harman, rational change in view is a complex process of balancing *conservatism* and *coherence*. The former aims to conserve as much as we can of what we currently accept; the latter aims to *increase coherence* (e.g., explanatory coherence, simplicity, etc.) and *decrease incoherence*. Exactly how to cash out 'coherence' and 'incoherence' is a difficult (and ongoing) issue; but the general idea is clear enough. One notable sort of *incoherence* is often tied to logical (negation-) inconsistency: rationality instructs us to *reject (logical) contradicitons* – reject any sentence (or proposition, etc.) of the form $A \wedge \neg A$. But even this sort of principle needs to be balanced with the pursuit of increasing coherence. It may be, for example, that glut theorists are right: given conservativeness with respect to (say) truth principles or the like, the most coherent (e.g., most explanatory, simplest, etc.) response to standard antinomies (e.g., liar paradox) takes them to be gluts.[6] But such is the messy – and 'defeasible' or take-back – life of rational inquiry: a balance between conservativeness and coherence.

3 Two traditional role(s) of logic

As above, Harman gives us a general distinction between logic and rational theory change. Taking this lesson on board raises a question: what role does logic play in rational theory change – or, more generally, in theory selection?

On this question there may be many answers.[7] For present purposes, two traditional roles of logic are important: closure and constraint. These correspond to two familiar faces of logical consequence: single- and multiple-conclusion consequence. I first review the two faces of consequence, and then take up the link between logic and rational change in view.

[6]Harman doesn't entertain this possibility, but, interestingly, he does point to antinomies as examples of data that can make for complexity of rational change in view [28].

[7]Harman [29] discusses a number of concrete roles that logic can play in rational change in view. Nothing I say below is intended to be in significant conflict with Harman's discussion, though my concern is not to stay true to details of Harman's work.

3.1 Closure: logic as one

Logic is most familiar in its so-called single-conclusion guise:

DEF. X *implies* A (notation: $X \vdash A$) iff there's no 'possibility' (or model, or etc.) that satisfies X but dissatisfies A.

Intuitively, a sentence A is satisfied by m (a model, possibility, context, whatever) just if m 'makes true' the sentence A.[8] In turn, a set X of sentences (theory X) is satisfied by m iff everything in X is satisfied by m.[9]

Logic, so understood, delivers an important theory: the *closure* of the theory you give it [49].

SC. $Cn(X) = \{A : X \vdash A\}$.

Give to logic your theory X, and then sit back: logic 'freely' or 'automatically' expands your theory to $Cn(X)$, which contains all of X's (singleton) consequences.

3.2 Closure: logic as many

There is a more general guise that logic takes, its so-called multiple-conclusion guise [34, 48]. We say that a theory X is *dissatisfied* by (model, possibility, etc.) m iff m dissatisfies everything in X. Consequence (logic) is then defined as before, but now over theories (sets of sentences) in general:

DEF. $X \vdash^{+} Y$ iff no 'possiblity' (or etc.) satisfies X and dissatisfies Y.

Single-conclusion consequence falls out as a special case, namely, one in which we identify singleton theories with their elements.

This 'general face of logic' delivers an analogous sort of closure set:

MC. $Cn^{+}(X) = \{Y : X \vdash^{+} Y\}$.

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[8] For present purposes, I leave things at this intuitive level. There are lots of ways this 'makes true' relation can be (and has been) cashed out formally. See Appendix for one example (where, e.g., the account of satisfaction is a cashing out of *makes true*), etc.

[9] I should remind that, given the topics under discussion (e.g., roles of logic in theorizing, etc.), I'm following the use of 'theory' that doesn't require theories to be closed under a logic. Such closure services is one traditional role of logic. This usage is common in philosophy, though the other usage (according to which all theories are closed theories) is more common in logic. (I'm grateful to an anonymous referee for prompting this footnote.)

Of course, $Cn^+(X)$ is not a theory; it is a set of theories – namely, all those theories that, according to logic, follow from X.

The question is: what role, beyond simply delivering closure sets, does logic serve in rational change in view (rational theory expansion, selection, etc.)? What, in short, is the link between rational theory change and logic?

4 The link: logic's constraint role

For convenient terminology, let us say that to *accept (theory)* X is to totally accept X, that is, to accept everything in X. Similarly, let us say that to *reject (theory)* X is to totally reject X, that is, to reject everything in X.[10]

What is the role of theory $Cn(X)$ in rational change in view? Think of $Cn(X)$ as the ideal: in *ideal circumstances* – perhaps not subject to the vicissitudes of balancing coherence and conservatism – you *ought (rationally) accept* $Cn(X)$ if you accept X itself. But this is (at best) the ideal.

Real change in view, as Harman emphasizes, is a complicated process of balancing coherence and conservatism; and quite often the conservatism–coherence balance, combined (for example) with limited cognitive capacities, can weigh against actually (versus ideally) accepting $Cn(X)$. As a result, a weaker connection has traditionally emerged.

The traditional link between rational theory change and logic is a familiar one: it's a condition on rational acceptance-rejection behavior. In its familiar (singleton-conclusion) guise, the link is roughly this:

r1. If $X \vdash A$, then it's irrational to accept X and reject A.

Of course, logic, in its closure role, will put A into the closure theory $Cn(X)$; and, as above, rationality may well demand that, at least in ideal circumstances (whatever they are, and if you're ever in them), you accept $Cn(X)$ if you accept X. But the weaker constraint, reflected in r1, is in play even in less-than-ideal circumstances (real change in view, so to speak); it requires only that you ought (rationally) *not* reject A from your theory X, given that, according to logic, X implies A.

[10] This usage of 'accept X' and 'reject X' is only an abbreviation that affords convenience; it isn't intended to suggest that all – or even the most ordinary of – uses of 'accept X' or of 'reject X' collapse into the given 'totally accept' or 'totally reject' uses. (If the reader prefers, s/he may simply insert the 'totally' explicitly.)

And here is where $Cn^+(X)$, the more general closure set (of theories), becomes relevant. In particular, logic's *constraint role*, in rational change in view, is nicely represented in its general (multiple-conclusion) form:

R. If $X \vdash Y$, then it's irrational to accept X and reject Y.

Suppose that you accept X. The upshot of R is that, by rationality's lights, you ought not reject (everything in) Y, given that, according to logic, X implies Y.[11]

What is important, for present purposes, is what logic does *not* do: logic, in its constraint role, doesn't tell us which, among theories in $Cn^+(X)$, you must accept, reject, etc. Indeed, logic is silent on which of such given logically available options are *rational options*. Logic constrains the space of rationally available options – nothing more. This is the gist of R.

As with r1, which R generalizes,[12] R leaves a lot of room: it is a negative condition; it proscribes certain theory combinations from rational options for acceptance and rejection.

The freedom in R is directly relevant to the main challenge concerning detachment freedom (see §1). I argue below (see §6) that the freedom afforded by R, in concert with freedom afforded by logic itself (more below), provides the answer to our guiding challenge – making sense of life without detachment (i.e., life without the validity of modus ponens). But first I sketch, very briefly, broadly and informally, some background on gluts and the failure of detachment.

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[11] Logic's traditional constraint role, represented in multiple-conclusion guise, has been put to new work towards an inferentialist-meaning account of logical consequence by Restall [44] and, in different directions, Ripley [45]. My own thinking on all of this has benefited from their work, and especially from conversations with them. One important note of difference: while all of us (even traditionally) agree on the basic roles of logic as constraint on rational theory selection, the novelty of (and controversy in) Restall's work lies in his taking consequence (validity, logic) to be *defined* via that role – defined in relation to our theory-selection practices (or, as he sometimes puts it, practices of assertion/denial). I resist such links, seeing logic more traditionally: logic affords an important constraint on theory-selection (or rational change-in-view) practices, but it's not defined in terms of such practices.

[12] Take Y to be $\{A\}$, the special singleton-theory case.

5 Antinomies and detachment

I believe that (material) modus ponens (detachment) is invalid: there are relevant possibilities where $\{A \supset B, A\}$ is satisfied (true) and B not. Here, and throughout, $\supset$ is the material conditional, defined per usual via negation and disjunction: $A \supset B$ is defined to be $\neg A \vee B$.[13]

Why think that detachment, so understood, is invalid? The best example, going back to Asenjo [3], Asenjo–Tamburino [5], Dunn [21, 22], Routley [46] and, most explicitly, Priest [35], involves a liar-paradoxical sentence which is both true and false – it is true and its negation is also true. Let L be such a sentence.[14] We have it that not only is L true, but so too $\neg L$ is true. Hence, since disjunctions are true if at least one disjunct is true, $\neg L \vee B$ is true for any B. But, now, where B is untrue, we have a counterexample to modus ponens: L is true; $\neg L \vee B$ and, hence, $L \supset B$ is true; but B not.

My interest here is not to argue the details of the 'glutty' treatment of antinomies. That case is made in various places [6, 36], and debate continues on [18, 24, 25, 26, 27, 37]. My chief interest is rather to show, on the basis of the foregoing framework (combining Harman's basic point and the traditional roles of logic), why the absence of modus ponens is neither a radical nor problematic proposal.

6 Towards detachment-free logic

While my proposal is compatible with a variety of 'glutty logics',[15] I focus on a particular logic: namely, LP^+, a natural (multiple-conclusion) sub-classical logic which accommodates gluts. The formal details are relegated to an appendix. The basic idea can be seen via a few informal remarks that zero

[13]An equally invalid argument form is disjunctive syllogism: $\{\neg A, A \vee B\} \therefore B$. Throughout, I focus on the detachment (modus ponens) form, despite equivalences delivered by negation and disjunction behavior.

[14]One might think that if our T-biconditionals, invoked in the standard liar-paradoxical derivations, are material biconditionals (with $\equiv$ defined via conjunction per usual), and such biconditionals are non-detachable, we don't get gluts after all. That's wrong (at least in the proposed logic); one does get such gluts, as $L \equiv \neg L$ is equivalent, by definition, to $(\neg L \vee \neg L) \wedge (L \vee \neg\neg L)$, which, in the target logic(s), implies $\neg L \wedge L$.

[15]Indeed, I believe that the proposal is directly related to non-glutty (non-paraconsistent) logics, and a variety of well-known non-classical (sub-classical) logics that are 'live options' in philosophy today. But I focus, for simplicity, on the case closest to my own views [6, 8].

in on the target connectives: negation and disjunction (out of which the material conditional is constructed). In particular, we give both truth and falsity conditions for such connectives. We assume that, on any 'possibility' or model m, every sentence A is (at least) true or (at least) false.[16] With this in hand we give truth and falsity conditions thus:[17]

- $\neg A$ is (at least) true iff A is (at least) false;
- $\neg A$ is (at least) false iff A is (at least) true.

- $A \vee B$ is (at least) true iff A is (at least) true or B is (at least) true;
- $A \vee B$ is (at least) false iff A is (at least) false and B is (at least) false.

Satisfaction of a sentence is its being at least true; and theory (set) satisfaction is all-elements satisfaction. *Dis*satisfaction of a sentence is its being untrue – not being even at least true (being just false, so to speak) – and theory (set) dissatisfaction is all-elements dissatisfaction. Consequence, in turn, is defined as before:

Def. $X \vdash^+ Y$ iff there's no possibility (model, whatever) in which X is satisfied and Y dissatisfied.

The resulting (sub-classical) logic LP$^+$ is 'paraconsistent'. In particular, negation-inconsistent (glutty) theories do not 'explode' into triviality: $\{A, \neg A\} \nvdash^+ \{B\}$.[18] A counterexample involves any A such that A and $\neg A$ are at least true (and, hence, A is at least false), and any just-false B. On any such possibility, the foregoing truth and falsity conditions deliver that $\{A, \neg A\}$ is satisfied; and so $\{A, \neg A\} \nvdash^+ \{B\}$. Moreover, and more to the

[16] If one prefers, one can, as Dunn pointed out [20, 21], think of non-functional relations ρ that relate all sentences A either to 1 or to 0. If A stands in ρ to 1, we say that A is *at least true* (on ρ, though this is often left implicit). If A stands in ρ to 0, we say that A is *at least false* (on ρ, but this is often left implicit). Important: there is no requirement that bars a sentence from being a glut; we can (though needn't) have both $A\rho 1$ and $A\rho 0$.

[17] The ideas for these clauses, at least in their modern guise, go (at least) back to Asenjo [3] and Asenjo & Tamburino [5], but the clauses I give here are due chiefly to Dunn [20, 21, 22]; they are a variation on his algebraic semantics [1, §18] for target logics. See also Belnap [13, 14], who provides an influential discussion of a closely related logic (viz., FDE, of which LP$^+$ is a strengthening).

[18] This is essential for glut theories; otherwise, the rationality constraint R would make it irrational to reject any sentence!

principal issue, such a counterexample also establishes the invalidity of detachment, as the discussion in §5 shows:

$$\{A, A \supset B\} \nvdash^{+} \{B\}.$$

That modus ponens is invalid appears to many to be both implausible and (perhaps thereby) a devastating blow to the idea that there are gluts (or even relevantly possible gluts, etc.).[19] What I wish to show is that things are not as implausible or dire as they appear. Modus ponens is indeed invalid (given gluts, which we are assuming for present purposes); but this is not a stumbling block to our theoretical pursuits. What need to be highlighted are the common choices that logic itself leaves us, choices to be resolved in the common course of carrying out rational change in view.

7 Doing without detachment

Modus ponens, as above, is invalid. Many have thought that when we 'do modus ponens' or 'follow modus ponens', we are simply inferring (say, accepting something new) *in accord with logic*, that is, *in accord with what our theory logically implies*. Since $\{A, A \supset B\}$ doesn't imply B, we can't be 'in accord with logic' when we infer B from $\{A, A \supset B\}$, when we accept B into our expanded theory. But we are close.

What is notable about the invalidity of modus ponens is that it corresponds to a closely related validity. Specifically, while detachment is *in*valid,

$$\{A, A \supset B\} \nvdash^{+} \{B\}$$

a close cousin of detachment is *valid*:

$$\{A, A \supset B\} \vdash^{+} \{B, A \wedge \neg A\}.$$

[19] As noted in §1, an alternative view is to go on the quest for suitable conditionals, which often utilizes pioneering ideas of Anderson and Belnap [1, 2] and others in the relevance tradition. (NB: the best known such relevance logic R will not work for glutty theories, due essentially to Curry's paradox, as noted by Dunn, Meyer, and Routley [33]. But the quest has nonetheless continued by going weaker than Anderson and Belnap logics to 'basic relevant logics' explored by Meyer [32], Routley/Sylvan [47], Bimbó & Dunn [16], Dunn [20], Mares [31], Priest [40], Restall [43], Beall [6], and, among others, perhaps especially Brady [17].) Such a 'quest' is the dominant tradition, and has been much-discussed; my aim here, as noted in §1, is to advance an alternative – and, I believe, simpler – approach.

To see this, note that any counterexample to $\langle\{A, A \supset B\}, \{B\}\rangle$ is one in which A is a glut – it and its negation are at least true. Assuming, as is the case in LP^+, that a conjunction is (at least) true iff both conjuncts are (at least) true, such counterexamples are, one and all, cases in which $A \wedge \neg A$ is at least true. In general, there is no way to satisfy $\{A, A \supset B\}$ and also *dis*satisfy $\{B, A \wedge \neg A\}$. Any possibility in which $\{A, A \supset B\}$ is satisfied is one in which at least one of B and $A \wedge \neg A$ is satisfied.

·· *Parenthetical note.* I should note that, at least in the simplest case of LP^+, one can focus on single-conclusion variants of the multiple-conclusion cases. (I'm grateful to an anonymous referee for prompting clarification on this point.) In particular, given background (so-called structural) features of LP^+ together with the clauses for disjunctions, we get the validity of

$$A, A \supset B \therefore B \vee (A \wedge \neg A)$$

corresponding to the target 'choices validity' case (see below): namely,

$$\{A, A \supset B\} \vdash^+ \{B, A \wedge \neg A\}.$$

But this correspondence need not always hold for various non-classical logics, and may even disappear when the language becomes more involved than the simple target cases of LP^+ truth (or property, etc.) theories. In general, the 'choices' idea, to which we turn below, seems to be more intuitive when presented in the more general multiple-conclusion guise; and so I have followed that course here. *End note.* ··

7.1 Choices that logic leaves us

So what? What does this have to do with our practice of 'doing modus ponens' or 'following modus ponens'? By way of the answer, consider the sense, alluded to above, in which logic itself has left us with choices.

Let us say that a pair of theories $\langle X, Y\rangle$ is a *choices validity* iff logic says that it's valid but X fails to imply any proper subtheory of Y.[20] An important example, highlighted above, is the cousin of detachment:

$$\{A, A \supset B\} \vdash^+ \{B, A \wedge \neg A\}.$$

[20]In other words, $\langle X, Y\rangle$ is a *choices validity* iff $X \vdash^+ Y$ but there's no (proper) $Z \subset Y$ such that $X \vdash^+ Z$. (If one prefers, one can think of $\langle X, Y\rangle$ as a 'generalized argument', with X the premise set and Y the 'conclusion set'. Nothing hangs on this terminology.)

The given pair of theories is valid; but the first theory fails to imply any proper subtheory of the second. Evidently, logic has left us with a choice. You ask logic: what follows from the theory $\{A, A \supset B\}$? Logic, getting to the point, tells you that $\{B, A \wedge \neg A\}$ follows from $\{A, A \supset B\}$. And with that, logic has had its say; and logic has thereby left a choice.

Invoking the traditional rationality link R (see §4), the choice may be seen thus: since, according to logic, $\{A, A \supset B\} \vdash^{+} \{B, A \wedge \neg A\}$, it is irrational to accept $\{A, A \supset B\}$ and reject $\{B, A \wedge \neg A\}$. Suppose that you accept $\{A, A \supset B\}$, and that you want to expand your theory 'according to logic' – you want to add to your theory, not merely avoid logically clashing with it. In the case at hand, logic itself leaves you with the choice between adding B and adding $A \wedge \neg A$ (and, for that matter, adding both). Hence, since logic fails to deliver either B or $A \wedge \neg A$, you can't look to logic to make the choice. Where, then, do you look to make your (rational) choice? My suggestion is that you look where you always look: principles concerning rational rejection.

7.2 Principles of rational rejection

There is a strong, fundamental principle of rational change in view, which falls under the 'avoid incoherence' category. The principle, baldly stated, concerns explicit inconsistencies (see §2):

IR. Reject contradictions (i.e., sentences of the form $A \wedge \neg A$)!

This is but one of many variations of the rejection principle; but it is a familiar and uncontroversial one. We reject inconsistencies when we see them; and we do so without blinking.

My suggestion is that IR, perhaps in concert with similar rejection principles, governs much of what is normally called 'following modus ponens'. In particular, when we infer B from $\{A, A \supset B\}$, we are not, contrary to common opinion, *in accord with logic*; logic tells us only that $\{B, A \wedge \neg A\}$ follows from $\{A, A \supset B\}$; logic thereby leaves us with the choice. What we are doing, when inferring B from $\{A, A \supset B\}$, is actually choosing B via a rejection of $A \wedge \neg A$. Logic tells us that $\{B, A \wedge \neg A\}$ follows from our theory $\{A, A \supset B\}$; we rely on IR; we reject $A \wedge \neg A$; we accept B. That's the route of our rational change in view.[21]

[21]This sort of idea finds roots elsewhere [15, 35, 36], but its promise is insufficiently appreciated. By drawing on Harman's general framework and the traditional constraint role of logic, I hope to have made the picture simpler and the promise more visible.

Of course, IR is defeasible, like any principle concerning rational changes in view. That's the hard life of rational inquiry. According to glut theorists, our efforts at balancing the pursuits of conservativeness and coherence tilt against some applications of IR. In particular, the best balance of conservativeness and coherence has us *accepting* certain contradictions – the bizarre and, fortunately, rare ones like liar-paradoxical sentences. This isn't a hard knock against IR; it continues in full force for the vast array of normal cases. And such force is sufficient, in the vast array of normal cases, to get us to accept B from $\{A, A \supset B\}$ via a rejection of $A \wedge \neg A$.

7.3 Generalization

There is a result that encapsulates the relationship between classical logic (and its constraints on available theory selection) and the logic LP^+ under discussion. The exact details, available elsewhere [7], are less important than the general lesson drawn from the relationship.[22]

To see the result, call $p \wedge \neg p$ the *p-inconsistency claim*. Let $\iota(X)$ be the 'atomic inconsistency set' for X, which contains all p-inconsistency claims for every atomic subsentence p occurring in X. Then, where $\vdash_c^+$ and $\vdash_{lp}^+$ are the classical and (sub-classical) LP^+ logics, respectively, the result is this:

$$X \vdash_c^+ Y \text{ iff } X \vdash_{lp}^+ Y \cup \iota(X).$$

In short, classical logic and LP^+ agree, unless there is some inconsistency in the premise set (or 'first theory'). In effect, classical logic simply ignores $\iota(X)$, treating it as irrelevant to what follows from X. Glut theorists think that classical logic goes too far: it ignores important, however rare, negation-inconsistent theories in the space of logically possible theories. Rather than being irrelevant, $\iota(X)$ is an important factor in the space of logically available theories.

The upshot for our *inferences* from certain non-empty theories is significant: provided that we're prepared to reject $\iota(X)$, we can 'follow classical logic' in its record of implication, and in turn utilize R in our logic-informed changes in view. According to glut theorists, it isn't *logic* that removes all elements of $\iota(X)$ for us; *we* have to do that via *extra-logical* resources – principles of rational rejection or the like. This is what we are doing, and what

[22]The result can be generalized to the more general case of 'first-degree (or tautological) entailment' FDE^+, of which both LP^+ and the Kleene $\mathrm{K3}^+$ are proper extensions [8].

we have been doing, in cases of apparently 'applying modus ponens'. Logic doesn't sanction the detachment step (even if, contrary to fact, logic were to tell us what to accept or reject); extra-logical rejection principles – on which we all largely agree – are behind the step. We never had valid detachment; we simply relied on rational rejection principles to reject $\iota(X)$ and thereby choose B from $\{A, A \supset B\}$.

8 Summary and closing remarks

A common challenge to (standard sub-classical) glut theories is that they are implausibly weak: modus ponens is invalid. The challenge is to make it plausible that we can (and do) successfully carry on rational inquiry despite the invalidity of modus ponens. Towards meeting this challenge I have invoked a few old but good ideas: the general logic-reasoning framework advanced by Harman [29] and the traditional roles of logic, understood especially in multiple-conclusion terms. Combining these ingredients with common rejection principles and with the observation that logic, suitably understood, leaves us with 'choices' (in the form of choices-valid cases), delivers a plausible response to the challenge.

Even apart from the issue of gluts, the viability of a detachment-free language is intriguing. My hope is that I've shown a way in which freedom from detachment is indeed viable.[23]

[23] I am grateful to a great many philosophers who have contributed to my thinking on this topic. Along these lines, special thanks go to Hartry Field, Gilbert Harman, Edwin Mares, Vann McGee, Graham Priest, Stephen Read, Greg Restall, Dave Ripley, and Stewart Shapiro. Beyond these, I have greatly benefited from various discussions with Andrew Bacon, Phillip Bricker, Colin Caret, Roy Cook, Aaron Cotnoir, Susanne Bobzien, Justin D'Ambrosio, Michael De, Antony Eagle, Elena Ficara, Branden Fitelson, Salvator Florio, Patrick Girarad, Michael Glanzberg, Volker Halbach, Ole Hjortland, Leon Horsten, Michael Hughes, Dom Hyde, Dirk Kindermann, Hannes Leitgeb, Oystein Linnebo, Toby Meadows, Julien Murzi, Toby Napoletano, Andrew Parisi, Charles Pigden, Agustin Ráyo, Marcus Rossberg, Jeremy Seligmann, Lionel Shapiro, Noah Sharpsteen, Reed Solomon, Koji Tanaka, Henry Towsner, Zoltán Gendler Szabó, Zach Weber, and Bruno Whittle. I am also very grateful for encouraging and helpful feedback from audiences at Aberdeen (NIP), Alberta, Auckland, Connecticut (Logic Group), CUNY Grad Center, Glasgow, Lehigh University, Melbourne, Minnesota, Münich (MCMP), Ohio State (SEP), Otago, Konstanz (GAP), Princeton, Queensland, St Andrews (Arché), Sydney (SCFS), Wellington, Wollongong, and Yale (NELLC).

Appendix: review of LP^+

This appendix gives the briefest of sketches of the multiple-conclusion logic LP^+, defined model-theoretically. For present purposes, I restrict attention to the propositional level. Fuller details are available elsewhere [7, 8].

A Syntax

The syntax is that of classical propositional logic (CPL), taking (unary) $\neg$ and (binary) $\vee$ to be primitive, defining the other standard (boolean) connectives as usual (e.g., $A \wedge B$ is $\neg(\neg A \vee \neg B)$, etc.).

B Models (or 'semantics')

A natural approach to formally modeling the clauses for negation and disjunction is due to J. Michael Dunn [20, 21, 22]. In particular, we let our set of 'semantic values' be $\wp(\{\mathsf{t}, \mathsf{f}\}) \setminus \{\emptyset\}$, that is, the powerset of the standard two-valued set of 'semantic values' minus the emptyset. In turn, models are all and only those (total) functions $v : \text{Sentences} \longrightarrow \wp(\{\mathsf{t}, \mathsf{f}\}) \setminus \{\emptyset\}$ that obey the following clauses:

- $\mathsf{t} \in v(\neg A)$ iff $\mathsf{f} \in v(A)$.
- $\mathsf{f} \in v(\neg A)$ iff $\mathsf{t} \in v(A)$.

- $\mathsf{t} \in v(A \vee B)$ iff $\mathsf{t} \in v(A)$ or $\mathsf{t} \in v(B)$.
- $\mathsf{f} \in v(A \vee B)$ iff $\mathsf{f} \in v(A)$ and $\mathsf{f} \in v(B)$.

We let $\mathbb{V}$ be the set of all such models (or 'valuations', if you prefer).

Let A be any sentence. We say that A is *satisfied by (or on, according to, etc.)* v iff $\mathsf{t} \in v(A)$, and *dissatisfied* otherwise – that is, iff $\mathsf{t} \notin v(A)$, iff $v(A) = \{\mathsf{f}\}$.

We say that a theory (or set of sentences) X is satisfied by (on, etc.) v iff v satisfies everything in X. Likewise, we say that v *dissatisfies* a theory X just if v dissatisfies everything in X.

C Consequence

The consequence relation (validity, implication, entailment, etc.) is defined in the usual multiple-conclusion way:

Definition (LP$^+$ validity) $X \vdash_{lp}^{+} Y$ iff there's no $v \in \mathbb{V}$ that satisfies X but dissatisfies Y.

D Notable features

I note a few features that are relevant to the topic of the paper.

D.1 Paraconsistent

The logic is 'paraconsistent' in the sense that it invalidates *explosion*:

$$\{A, \neg A\} \nvdash_{lp}^{+} \{B\}.$$

Counterexample: take any $v \in \mathbb{V}$ on which A is glutty but B untrue, that is, any v such that $v(A) = \{\mathsf{t}, \mathsf{f}\}$ and $v(B) = \{\mathsf{f}\}$. Such a model satisfies $\{A, \neg A\}$ and dissatisfies $\{B\}$.

D.2 True detachment freedom

The counterexample(s) in §D.1 suffice to invalidate detachment (modus ponens) in material-conditional form:

$$\{A, A \supset B\} \nvdash_{lp}^{+} \{B\}.$$

Indeed, one can show that there is *no non-vacuous detachable connective* in LP$^+$ [11]; this is 'true detachment freedom'.[24]

D.3 Classical logical truths

Finally, one can show, via minimal tweaks on Priest's single-conclusion proof [35],[25] that LP$^+$ agrees with classical logic on all logical truths: whatever

[24]A binary connective $\odot$ detaches just if $\{A, A \odot B\}$ implies $\{B\}$, and does so non-vacuously only if $\{A \odot B\}$ fails to imply $\{B\}$. (Conjunction vacuously detaches.)

[25]See too Asenjo–Tamburino's single-conclusion proof [5].

classical logic claims is true 'in virtue of logic', LP^+ agrees; and the converse also (obviously) holds.[26] For those, like Quine [41, xi], who see logic as 'the systematic study of logical truths' (versus the theory or 'systematic study' of implication or consequence, generally), we have agreement: classical logic gets the science right. Where classical logic goes wrong is in its judgements about which theories follow from certain non-empty (in particular, negation-inconsistent) theories. This relationship between classical logic and LP^+ is reflected in the result noted in §7.3.

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[26] It's obvious when you notice that the set of classical models is a proper subset of $\mathbb{V}$. Hence, if nothing in $\mathbb{V}$ dissatisfies a sentence A, then so too for the corresponding set of classical models.

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