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LOGICAL PLURALISM

JC Beall and Greg Restall

'. . .[I]n considerations of a general theoretical nature the proper concept of consequence must be placed in the foreground.'

Alfred Tarski

I. Logic, Logics, and Consequence

Anyone acquainted with contemporary Logic knows that there are many so-called logics.[1] But are these logics *rightly* so-called? Are any of the menagerie of non-classical logics, such as relevant logics, intuitionistic logic, paraconsistent logics or quantum logics, as deserving of the title 'logic' as classical logic? On the other hand, is classical logic really as deserving of the title 'logic' as relevant logic (or any of the other non-classical logics)? If so, why so? If not, why not?

Logic has a chief subject matter: Logical Consequence. The chief aim of logic is to account for consequence, to say, accurately and systematically, what *consequence* amounts to, which is normally done by specifying which arguments (in a given language) are *valid.* All of this, at least today, is common ground.

Logic has not always been seen in this light. Years ago Logic was dominated by the Frege-Russell picture which treats logical truth as the lead character and consequence as secondary. The contemporary picture reverses the cast: *consequence* is the lead character. For example, Etchemendy writes:

> Throughout much of this century, the predominant conception of logic was one inherited from Frege and Russell, a conception according to which the primary subject of logic, like the primary subject of arithmetic or geometry, was a particular body of truths: logical truths in the former case, arithmetical or geometric in the latter. . . . This conception of logic now strikes us as rather odd, indeed as something of an anomaly in the history of logic. We no longer view logic as having a body of truths, the logical truths, as its principal concern; we do not, in this respect, think of it as parallel to other mathematical disciplines. If anything, we think of the consequence relation itself as the primary subject of logic, and view logical truth as simply the degenerate instance of this relation: logical truths are those that follow from *any* set of assumptions whatsoever, or alternatively, from no assumptions at all. [17, p. 74][2]

1 Except where grammar dictates otherwise 'Logic' names the discipline, and 'logic' names a logical system.

2 For a more detailed discussion of the centrality of *consequence* in logic see Chapter 2 of Stephen Read's *Thinking About Logic* [40].

But what is logical consequence? What is it for a conclusion, A, to logically follow from premises Σ? There is a tradition to which almost everyone subscribes. According to this tradition the nature of logical consequence is captured in the following principle:[3]

(V) A conclusion, A, *follows from* premises, Σ, if and only if any case in which each premise in Σ is true is also a case in which A is true.

Here is one example of the use of this principle to introduce validity. The quotation is taken from Richard Jeffrey's text, *Formal Logic: Its Scope and Limits.*

> Formal logic is the science of deduction. It aims to provide systematic means for telling whether or not given conclusions follow from given premises, i.e., whether arguments are valid or invalid. . . .
>
> Validity is easily defined:
>
> A *valid* argument is one whose conclusion is true in every case in which all its premises are true.
>
> Then the mark of validity is absence of counterexamples, cases in which all premises are true but the conclusion is false.
>
> Difficulties in applying this definition arise from difficulties in canvassing the cases mentioned in it. . . . [20, p.1]

Notice that, despite its familiarity, (V) does not give us a *complete* account of logical consequence. To construct a *logic* we need an accurate and systematic account of which arguments are valid. (V) by itself does not give us an account of the *cases* involved. Jeffrey's last line is significant: 'Difficulties in applying this definition arise from difficulties in canvassing the cases mentioned in it.' In this paper we present a view that takes such 'difficulties' very seriously. The view is *logical pluralism*—'pluralism', for short. Pluralism, we believe, makes the most sense of contemporary work in Logic.

II. Pluralism in Outline

To be a pluralist about logical consequence, you need only hold that there is more than 'one true logic'. There are hints of pluralism in the literature in philosophy of logic, but it has not been given a systematic sympathetic treatment.[4] In this paper we wish to introduce and defend a particular specific version of logical pluralism. This pluralism comes with three tenets:

(1) The pretheoretic (or intuitive) notion of consequence is given in (V).

[3] We read '(V)' as 'vee' for validity, not the Roman numeral five.

[4] The most extensive treatment to date is given by Resnik [41]. But even in Resnik's systematic essay the focus is primarily on non-cognitivism about logical consequence, an issue orthogonal to the concerns of this paper.

(2) A *logic* is given by a *specification of the cases* to appear in (V). Such a specification of cases can be seen as a way of spelling out *truth conditions* of the claims expressible in the language in question.

(3) There are *at least two* different specifications of cases which may appear in (V).

Point (1) is self-explanatory: Using (V) to determine logical consequence is by no means idiosyncratic. We will not attempt an extensive search of the literature, though evidence for the centrality of an analysis like (V) is not hard to find.[5] Logic is a matter of preservation of truth in all cases. This is the heart of logical consequence. However, this is not the end of the matter. To use (V) to construct a *logic* you need to spell out what these *cases* might be. To give a systematic account of logical validity, you need to give an account of the cases in question, and you need to tell a story about what it is for a claim to be *true in a case.*

Without an answer to these questions, you have not specified a logic. This truism is given in point (2) of our account of logical pluralism. To use (V) to develop a *logic* you must specify the cases over which (V) quantifies, and you must tell some kind of story about which kinds of claims are true in what sorts of cases. For example, you might give an account in which cases are *possible worlds.* (Furthermore, you might go on to tell a metaphysical story about what sorts of entities possible worlds are [24, 25, 49, 54].) On the other hand, you might spell out such cases as set-theoretic constructions such as *models* of some sort. However this is done, it is not the sole task. In addition, you must give an account of *truth in a case.*

Here is an example of how you might begin to spell this out. Your account of cases and truth in cases might include this condition, where A and B are claims and x is a *case*:

$A \wedge B$ is true in x iff A is true in x and B is true in x

Such an assertion tells us that a conjunction is true in a case if and only if both conjuncts are true in that case. This gives us an account of *truth* in cases which not only tells you how conjunction works, but it also gives you some data about validity. Once we have this connection, we have the validity of the argument from $A \wedge B$ to A. For any case x, if $A \wedge B$ is true in x then A is true in x, by the condition given above. This is but one example of how you might begin to systematically spell out the conditions under which claims are true in cases. To do this is to *do logic.*

None of this so far is particularly controversial.[6] The controversy in our position comes from point (3). According to the third and final claim there are *different* ways to specify

[5] Here is one more case: W.H. Newton-Smith, in his popular introductory text, writes that some arguments 'have true conclusions whenever they have true premises. We will say that they are *valid.* That means that they have the following property. In any case in which the premise (premises) is (are) true, the conclusion must be true' [34, p.2].

[6] Well, one part *is* controversial. We have privileged the *model-theoretic* or *semantic* account of logical consequence over and above the *proof-theoretic* account. A version of pluralism can be defended which does not privilege 'truth in a case'. However, most of the current debates with which we are interacting lie firmly within this model-theoretic tradition, and we are comfortable with that tradition, so we develop pluralism in this way.

the 'cases' appearing in (V). There is no *canonical* account of cases to which (V) appeals. There are different, equally good ways of spelling out (V); there are different, equally good *logics*. This is the heart of logical pluralism.

We will begin our elaboration of (3) by examining different ways (V) has been filled out. We start with a well-known way of filling out (V): Models for classical first-order logic.

III. Tarskian Models, and Classical Logic

There are many ways in which you might give an account of (V) which renders valid all of the theses of classical logic. One way is to treat the cases of (V) as *possible worlds*. Then your clauses for truth in a case, or truth in a world, will look like this.

$A \wedge B$ is true in w iff A is true in w and B is true in w.

$A \vee B$ is true in w iff A is true in w or B is true in w.

$\sim A$ is true in w iff A is not true in w.

It is a little harder to give an account of the truth of quantified claims in possible worlds, but if we allow each object in each world to have a *name* in our language, then the clauses are trivial.

$\forall x A(x)$ is true in w iff for each object b in w, $A(b)$ is true in w.

$\exists x A(x)$ is true in w iff for some object b in w, $A(b)$ is true in w.

Now, with no further analysis of what a world w might *be*, or how *many* there might be, a story of consequence can be told. We have already seen that this account validates the inference from $A \wedge B$ to A. It also validates the inference from A to $A \vee B$, from $A \wedge (B \vee C)$ to $(A \wedge B) \vee C$, from $\forall x(A \vee B)$ to $\forall x A \vee \exists x B$, and many more besides.

If the cases in our account encompass *all* possible worlds then an argument is valid if and only if in any world in which the premises are true, so is the conclusion, or equivalently, if it is impossible for each premise to be true but for the conclusion to not be true. Call this the *necessary truth preservation* account of validity. This is one way to elaborate (V), but it is not the only one. In fact, it is not at all the *traditional* picture of logical consequence. The possible worlds account is not *formal* because it makes no essential use of the *forms* of the claims analysed. To be sure, our elucidation has picked out conjunctions, disjunctions, negations and quantifiers, but there was no need at all to do this. We could just as well have given clauses for colour terms:

a is red is true in w iff a is red in w.

a is coloured is true in w iff a is coloured in w.

This explains why the necessary truth preservation account of validity renders the argument from *a is red* to *a is coloured* valid. The argument is valid because in any case (that is, in any possible world) in which something is red, it is also coloured. It is impossible that something be red but that it fail to be coloured.

This is not the only way to account for logical consequence, and, as we have mentioned, it is not the mainstream tradition. According to logical orthodoxy, the argument from *a is red* to *a is coloured* is invalid, because it is not *formal*. It does not exploit any logical form: it has the form $Fa \vdash Ga$, and this form is invalid. We can give an account of this form of validity by varying the cases over which (V) quantifies. Now validity is a matter of form, and cases interpret *formal* languages—in this example, the languages of first-order logic, in which we have simple predicates, names, variables, quantifiers and connectives. Sentences in such a formal language are interpreted in a *model*, *Tarskian* models of first-order logic. A Tarskian model, M, is a structure that comprises the following:

A *nonempty set D*, the *domain;* and

A *function I*, the *interpretation*, satisfying the following conditions:

$I(E)$ is an element of D, if E is a name (in the given language);

$I(E)$ is a set of ordered n-tuples of D-elements, if E is an n-place predicate.

Then we use a model to interpret the language.[7]

If α is an assignment of D-elements to variables, then $I_\alpha(x)=\alpha(x)$. If a is a name, $I_\alpha(a)=I(a)$.

$Ft_1 \ldots t_n$ is true in $\langle M,\alpha\rangle$ iff $\langle I_\alpha(t_1), \ldots, I_\alpha(t_n)\rangle \in I(F)$.

$A \wedge B$ is true in $\langle M,\alpha\rangle$ iff A is true in $\langle M,\alpha\rangle$ and B is true in $\langle M,\alpha\rangle$.

$A \vee B$ is true in $\langle M,\alpha\rangle$ iff A is true in $\langle M,\alpha\rangle$ or B is true in $\langle M,\alpha\rangle$.

$\sim A$ is true in $\langle M,\alpha\rangle$ iff A is not true in $\langle M,\alpha\rangle$.

$\forall xA$ is true in $\langle M,\alpha\rangle$ iff A is true in $\langle M,\alpha'\rangle$ for each x-variant α' of α.

$\exists xA$ is true in w iff A is true in $\langle M,\alpha'\rangle$ for some x-variant α' of α.

We take models to be cases, and we have defined truth in a model for sentences of a formal language, by the standard recursive clauses. This account then tells us about validity for arguments in the formal language, by way of (V). An argument is valid if and only if in every model in which the premises are true, so is the conclusion. For arguments of our natural language, validity is inherited by way of *formalisation*. We can define truth-in-a-model for claims of English by the standard processes of regimentation of those claims, and therefore we can define *validity* for natural language arguments. Call this account the *Tarskian* account of validity of arguments in natural language.

[7] We use assignments of values to variables, in order to interpret sentences with free variables. If α is an assignment of values to variables, $\alpha(x)$ is the value of the variable x. Furthermore, an x-variant of α is an assignment which agrees with α in the values of all variables except possibly x.

We now have our first dimension of plurality. Consider the question: Is the argument from *b is red* to *b is coloured* valid? We have seen that the answer is *yes* for validity as necessary truth preservation. The answer is *no* for the Tarskian account of validity. This argument has the form $Fa \vdash Ga$, and there are many models in which the premise is true and the conclusion false. So, we have at least two different accounts of validity. One might now wonder: Is there any basis upon which to choose between these two accounts? Is there any reason you might prefer one to the other? The answer here is a resounding *yes*. Tarskian validity is *formal*; necessary truth preservation is not. Tarskian validity can (perhaps) be known a priori, but necessary truth preservation (probably) cannot. If Kripke is correct [22], the argument from *b is water* to *b is* H_2O is necessarily truth preserving, but this cannot be known a priori.

On the other hand, validity as necessary truth preservation does not rely on a choice of the family of logical constants. Colour connections, temporal, spatial and other modalities, part–whole relations, and many other forms of necessary connections are equally encompassed by this account. The Tarskian account, on the other hand, makes a choice of logical constants, the privileged parts of language which can contribute to logical form, and hence, logical validity. Not all Logic is simply a matter of form. (This is one part of Etchemendy's criticism of the Tarskian account of logical validity [18].)

A pluralist on the question of *formality* will call both accounts *logic*. Thoroughgoing pluralists will be happy to call the result of both Tarski's account, and the necessary truth preservation account,. *logic*, for both are ways of spelling out the pretheoretic account (V) of logical consequence. The proper answer to the question 'Is the argument from *b is red* to *b is coloured* really valid?' is to say '*Yes*, it is necessarily truth preserving, and *no*, it is not valid by first-order logical form.'

A pluralist account of disagreement about logical form goes as follows: It is not fruitful to debate which of these things is *logic*. Both flesh out (V), so *both* are logic. Given an argument which is necessarily truth preserving but not Tarski-style valid, it is surely more informative to say: Yes, there is no possibility in which the premise is true and the conclusion false, but there is a Tarski-style model in which the premise is true and the conclusion false, and this shows the necessary truth preservation is not in virtue of the first-order logical form of the claims involved. That is informative analysis. A debate about which of these is logic adds nothing.

However, this is not the only kind of problem people might have with the Tarskian analysis of logical consequence and first-order logic. Consider the zero-premise arguments to conclusions such as $\vdash \exists x(x=x)$ or $\vdash \exists x(Fx \vee {\sim}Fx)$. If cases comprise Tarskian-style models these arguments are valid. Famous debates have raged over this result. A long and rather formidable tradition claims that neither '$\exists x(x=x)$' nor '$\exists x(Fx \vee {\sim}Fx)$' is Really Valid; *logic*, in this tradition, allows for the empty case, but Tarskian-style cases are never empty.[8]

[8] Another famous objection is voiced by Kreisel [21], Boolos [9], and McGee [27], to the effect that the models given in the traditional Tarskian account of validity are too limited. Why not allow for domains too 'big' to be sets? Logic alone seems not to impose this restriction, but traditional Tarskian cases do.

There is also a philosophically illuminating independent justification for pluralism on the matter of the domain of quantification. Phillip Bricker [11] has developed an account of modal realism which deals with the 'isolated universes' problem by allowing not only concrete possible worlds as

The foregoing concerns fit a pattern in this way: Let *C* be an account of consequence, or some precisification of 'validity'. Then *C* is said to *undergenerate*, with respect to some argument, if that argument is Really Valid but not *C*-valid. The problem, in this case, is that *C* gets things wrong by failing to call the argument 'valid' when 'in fact' it is valid—*Really Valid*. *C* is said to *overgenerate*, with respect to some argument, if the argument is *not* Really Valid but *is* *C*-valid. In this case, *C* gets things wrong by calling the argument 'valid' when 'in fact' it is not.

The undergeneration–overgeneration pattern is ubiquitous in philosophy of logic; indeed, it may well be *the* central pattern of dispute in the field.[9] The important point here is that our pluralism can make sense of the debate, though in general it refrains from blessing *only one* side of the debate with the title 'logic'. In particular, a pluralist response to these issues goes as follows: Many appeals to 'Real Validity' are appeals to *real* validity; they are not, however, appeals to the *only* real validity. Real validity comes from a specification of cases which appear in (V). According to pluralism there are at least two such specifications of cases. So far, we have seen two different approaches within *classical* logic — the worlds approach, and the Tarskian models approach. But these are just the beginning.

IV. Situations, and Relevant Consequence

Each of the accounts of interpretations or truth conditions seen so far have been *classical* with respect to negation. For any cases x seen so far, be they worlds, Tarskian models, class-size models, or even models with empty domains,

$\sim A$ is true in x iff A is not true in x.

Call this the *classical negation clause*. There are many good reasons for using a classical negation clause in constructing an account of truth in cases. The most obvious reason is the way that we use negation, and the conditions under which negations are, in fact, true: $\sim A$ is true just when A is not true. This, one might say, is simply what 'not' *means*.[10] But to infer from this truism that the classical negation clause is the only one worth using in elaborating (V)'s cases would be far too swift. To do so would be to assume that the only acceptable use of cases is to model consistent, complete *worlds*. But many have questioned this assumption. There are other ways to give an account of cases, or conditions under which claims might be true or false. One such account is the *situation theory* of Barwise and Perry [1, 2, 3].

The world is made up of situations. They are simply *parts* of the world. Claims are true of not only the world as a whole, but some claims at least are true of situations. We will

[8] *continued*
units of evaluation, but also classes of possible worlds. A class of worlds does duty for a 'world' with spatio-temporally disconnected parts. Now, classes *may* be empty. If we allow the empty class, and we define validity as truth preservation in all *classes* of worlds, we have a free logic. If we do not, our logic has existential import. Which should you choose? The metaphysical view need not constrain you, according to pluralism.

[9] The terms 'undergeneration' and 'overgeneration' are found elsewhere [18, 39, 40].

[10] You might well say, instead, that this is what *true* means.

not spend time on the theory of situations and their individuation here: we will simply illustrate it. In the situation involving Greg's household as he writes this, it is *true* that Christine is reading a paper. It is also *true* that the stereo is playing. It is *false* that the television is on. It follows from this, and the fact that the television is in fact an inhabitant of the situation, that it is *true*, in this situation, that the television is off.

Situations 'make' claims true and they 'make' others false. However, some situations, by virtue of being *restricted* parts of the world, may leave some claims undetermined.[11] It is not true in this situation that JC is reading. It is also not false in this situation that JC is reading — that is, it is not true in this situation that JC is *not* reading. JC does not feature in this situation at all.[12]

It follows that the classical account of negation fails *for situations*. This treatment of negation is out of place in this context. It seems plausible, however to hold fast to the classical analyses of conjunction and disjunction.

$A \wedge B$ is true in s iff A is true in s and B is true in w.

$A \vee B$ is true in w iff A is true in w or B is true in w.[13]

We must emphasise at this point that the non-traditional treatment of negation does not mean that we are modelling a *non-classical* negation. Quite to the contrary. Our treatment of negation is not the traditional one simply because we are entering a new field — the logic of situations. It has not been traditional to formally model claims of the form 'A is true in situation x'; once you do so, and once you acknowledge that situations are restricted parts of the world, it becomes clear that you ought reject the classical treatment of negation when applied to situations. This is completely consistent with the classical treatment of the truth or falsity of negation simpliciter. We may maintain that $\sim A$ is true if and only if A is not true. That is not in question. The situation theoretic analysis of this equivalence will proceed further: $\sim A$ is true if and only if $\sim A$ is true in some (actual) situation or other. A is not true if and only if A is not true in any (actual) situation whatsoever. The traditional, classical equivalence is maintained if we agree, then, that if $\sim A$ is true in some (actual) situation, then A is not true in any (actual) situation. And this is simple to maintain, given three plausible theses:

(1) There is a situation, w, of which every actual situation is a part.

[11] We use shudder quotes around 'make' here not that we wish to avoid the use of *truthmaking* terminology. To the contrary, we value the recent revival of this terminology and the analysis of the connections between claims and parts of the world which make them true [4, 19, 32, 42]. However, this terminology is not used by situation theorists, and that it would be a mistake to impute it to them.

[12] Likewise, in the situation in which JC writes *this*, it is not true that Greg is thinking, and it is not false that Greg is thinking. The situation in which JC writes *this* does not involve Greg at all, in which case neither 'Greg is thinking' nor its negation enjoy the required 'truthmakers' in the situation at hand.

[13] The conjunction clause is never disputed; the disjunction clause is sometimes disputed. The given disjunction clause, however, seems sound for the intended interpretation. If in this situation the cat is on the table or in the cupboard, then either in this situation the cat is on the table or in this situation the cat is in the cupboard.

(2) If A is true in s and s is a part of s', then A is true in s'.

(3) If s is an actual situation in which $\sim A$ is true, then A is not true in s.

These theses connecting negation and situations ensure the truth of the classical account of negation. Negation, here, is classical.

The work, however, is not yet done; what is needed is a systematic treatment of the truth or falsity of negations in situations. This can be done in any of a number of ways. You can, for example, take satisfaction and dissatisfaction of relations in situations as primitive, and then inductively build up truth and falsity conditions of complex claims.[14] This approach is traditional in situation theory, and it is also used in some varieties of semantics for non-classical logics [2, 6, 14, 33]; however we will favour a different approach, a *compatibility* semantics stemming from Dunn's [15, 16, 43] analysis of negation.

On the approach we shall follow negation behaves in situations much like necessity or possibility does in possible worlds. We admit into our semantics non-actual situations (or *models* of non-actual situations) which are connected by a binary relation of *compatibility*, which we write 'C'. Given this apparatus negation is definable.

$\sim A$ is true in s iff for any s' such that sCs', A is not true in s'.

Accordingly, the negation $\sim A$ is true in s just when any situations in which A is true are incompatible with s. This clause follows fairly immediately from the meanings of negation and compatibility. If $\sim A$ is true in s and A is true in s', then s is not compatible with s'. Conversely, if A is not true in any s' compatible with s, then it appears that s has *ruled A out*. That is, $\sim A$ is true in s. This reading does not rely on a 'funny' negation; it is completely compatible with a classical view of negation.[15] Given such a semantics of situations a natural reading of (V) emerges: a *situated* reading.

The argument from Σ to A is *relevantly valid* if in any model, in any situation in which all premises in Σ are true, so is A.

To speak loosely but suggestively: To make the premises true you make the conclusion true too. The *relevance* of this reading of consequence is immediate. The inference from A to $B \vee \sim B$ fails, since a situation in which A is true need not be one in which $B \vee \sim B$ is true.

If we take the relevant *tautologies* to be those claims true in every situation, then $B \vee \sim B$ is not among them. This does not mean that we have adopted a strange non-classical account of negation. We agree with the classical theorists that $B \vee \sim B$ is true in *every* world, where worlds are (at least) complete. Our negation is classical. The argument from A to $B \vee \sim B$ is classically valid in that any (possible) world in which A is true is one in which $B \vee \sim B$ is true; the *invalidity* of the given argument is a *relevant* invalidity, as there are situations in which the premise but not the conclusion is true.

[14] For example, you will say that not only is a conjunction true in s when both conjuncts are true, but dually, a conjunction is false when one conjunct is false.

[15] The three minimal conditions cited earlier for a classical treatment of negation have their 'compatibility' readings. (1) Any actual s is a part of a world w (this is as before); (2) w is a world if and only if wCw, and if wCs then s is part of w (in other words, worlds are maximal, self-compatible situations); (3) if sCt, s' is a part of s and t' is a part of t, then $s'Ct'$ too (compatibility of wholes leads to compatibility of parts).

The move to situations as incomplete parts of the world is natural. Another natural, but perhaps more daring, step is to invoke not only incomplete situations but also *inconsistent* situations—or *ways things could not be* [29, 43, 44, 47, 55]. These are situations which fail to be self-compatible. If, for example, s is not compatible with itself, then it is possible that both A and $\sim A$ be true at s. This, again, is not terribly non-classical. According to our given account of worlds as consistent, complete situations, such *impossibilia* cannot be a part of any *world*. Worlds are consistent, and hence, have no inconsistent parts. This does not mean, of course, that there are no *ways that things could not be*; it means, simply, that the worlds are not (and could not be) among them.[16]

Given the admission of inconsistent situations, an argument from $A \wedge \sim A$ to B fails the relevant test; for a situation in which $A \wedge \sim A$ is true need not be one in which B is true. A situation might well be *inconsistent* about A without involving *everything*. This same situation gives us a counterexample to *disjunctive syllogism*, the argument from $A \vee B$ and $\sim A$ to B. A situation inconsistent about A but not judging B as true suffices; $A \vee B$ is true in this situation, as is $\sim A$, but B fails.

This last case has been the cause of much debate in the literature on relevant logics and relevant inference. Much ink has been spilled on the failure of disjunctive syllogism and whether it is a virtue or a vice [28, 38, 46]. We do not plan to add to the spilling of ink in any depth here. We will simply note that traditional criticisms of the relevant rejection of disjunctive syllogism are beside the point, when seen in the light of pluralism. We will end this section on relevant consequence by explaining why this is so.

One cause of concern with the rejection of disjunctive syllogism is that disjunctive syllogism is *obviously* valid, and we reason with it all the time—we could not do without it in everyday reasoning [7]. Our pluralism will agree: *Of course* there is a sense in which disjunctive syllogism is valid—and even *obviously* so. After all, any (possible) world in which the premises are true is one in which the conclusion is true; in *that* sense—the sense afforded by cases as worldlike (complete and consistent)—disjunctive syllogism is valid. The virtue of a pluralist account is that we can enjoy the fruits of relevant consequence as a guide to inference without feeling guilty whenever we make an inference which is not relevantly valid. With classical consequence you know you will not make a step from truth to falsehood, assuming, with most philosophers, that *possible* worlds are complete and consistent. With relevant consequence, the strictures are tighter; you know you will not make a step from one that is true in a situation to something not true in it (but which might be true outside it). This is a tighter canon to guide reasoning.[17]

So, the case of incomplete and inconsistent situations motivates a genuinely different elucidation of logical consequence—one which differs with the classical account on the validity of inferences down to the propositional level. This account of consequence is still recognisably *logic*; it is another way to flesh out our condition (V). It is not a *rival* in any

[16] One of us (Greg) endorses just this view of (possible) worlds; the other of us (JC) does not. Nonetheless, for ease of exposition we treat (possible) worlds as complete and consistent. Our differences on this matter are important; however, as we discuss briefly in §VI, our pluralism is neutral with respect to the issue.

[17] For more elaboration and defence of this point see 'Defending Logical Pluralism' [5].

sense to the classical, traditional explications of that condition. Instead, it coexists alongside classical validity as simply another important variety of logical consequence.[18]

We now have a richer pluralism, a pluralism in which (V)'s cases may be worlds, incomplete situations, and even incomplete and inconsistent situations. For now, we close by adding one more element to the variety; we explain how pluralism makes sense of *intuitionistic consequence* in terms of (V)'s cases.

V. Constructions, and Intuitionistic Consequence

Mathematicians do not, generally speaking, concern themselves with a *situated* account of logical consequence while reasoning about mathematical objects or structures. However, they too can make some distinctions which are blurred by classical accounts of validity. We have in mind the mathematics pursued by mathematical *constructivists*.

The constructivism of the mathematicians Errett Bishop [8, 9], Douglas Bridges [12], Fred Richman [31, 45] and others can best be described as mathematics *pursued in the context of intuitionistic logic*.[19] In constructive mathematics the goal is to gain understanding of mathematical structures and to prove theorems about them, just as in classical mathematics; however, the goal is to prove mathematical theorems with constructive, or computational content. If a statement asserting the existence of some mathematical object is proved in a constructive manner (using the rules of intuitionistic logic) then this proof will contain the means of specifying the object or structure in question. Wittgenstein illustrates the advantages of constructive proof over its classical cousin by drawing out its implications for our *understanding*:

> A proof convinces you that there is a root of an equation (without giving you any idea *where*)—how do you know that you understand the proposition that there is a root? [53, p. 146]

This feature of constructive mathematics is guaranteed by the structure of constructive proofs. We emphasise the fact that this is a new notion of proof by using the word 'construction' for this notion. Constructions obey the following laws:

A construction of $A \wedge B$ is a construction of A together with a construction of B.

A construction of $A \vee B$ is a construction of A or a construction of B.

A construction of $A \supset B$ is a technique for converting constructions of A into constructions of B.

There is no construction of $\perp$.[20]

[18] We have restricted our attention here to the conjunction, disjunction and negation fragment of relevant logics. More can be done to bring the notion of relevant entailment into the language. For another approach to relevant logics which motivates *two* varieties of consequence, but from a very different perspective, we refer the reader to Mark Lance's 'Two Concepts of Entailment' [23].

[19] Tait provides a more explicitly *philosophical* account which draws very similar distinctions to the work of constructive mathematicians [50, 51].

[20] We define $\sim A$ as $A \supset \perp$. Accordingly, a construction of $\sim A$ is a technique for converting a construction of A into absurdity; it shows that there are no constructions of A.

A construction of $\forall xA$ is a rule giving, for any object, n, a construction of $A(n)$.

A construction of $\exists xA$ is an object, n, together with a construction of $A(n)$.

This elucidation is *not* formal; it is informal in various respects, perhaps most notably in its leaving the central notion of a construction undefined.[21] For all its informality, however, the account gives us an understanding of the behaviour of constructive proof. For example, the inference from $\forall x(A \vee B)$ to $\exists xA \vee \forall xB$ is classically valid but not constructively valid. For example, it is easy to demonstrate that every string of ten digits in the decimal expansion of π is either a string of ten zeros, or it is not. This does not give us a construction of the claim that either there is a string of ten zeros in π or every string of ten digits in π is not a string of zeros; any construction of *this* claim must either prove that there is no string of ten zeros in π or to show where one such string is. The constructive content of $\exists xA \vee \forall xB$ is greater than that of $\forall x(A \vee B)$.

Theorems of constructive mathematics are simply theorems of mathematics proved constructively.[22] According to this approach, the theorems of constructive mathematics are also theorems of classical mathematics. The difference between constructive and classical mathematics is not one of subject matter, but one of the required standards of proof. Classical mathematicians may appeal to the law of the excluded middle, and proof by contradiction; constructive mathematicians do not, as these moves destroy constructivity.

A truth conditional semantics may be given for the intuitionistic logic of constructive mathematics, which both does justice to the practice of constructive mathematics and opens the way for a pluralist reading of that practice. The truth conditional semantics is simply Kripke's semantics for intuitionistic logic. Truth is relativised to *points* (which model constructions) which are partially ordered by *strength* (written '$\geq$').

$A \wedge B$ is true in c iff A and B are true in c.

$A \vee B$ is true in c iff A is true in c or B is true in c.

$A \supset B$ is true in c iff for any $d \geq c$, if A is true in d then so is B.

$\sim A$ is true in c iff A is not true in d for any $d \geq c$.

The points in a Kripke structure for intuitionistic logic do a good job of modelling *constructions* ordered by a notion of relative strength. The clauses for conjunction and disjunction are straightforward transcriptions of our pre-formalised notion of constructions. The rules for implication and negation differ somewhat, but can be motivated to

[21] Note the similarity, here, to the account of worlds at the start of §III. In §III we gave an account of what it is for a conjunction to be true in a world; however, we gave no account of what it is for an arbitrary claim to be true in a world. Similarly here, we give no account of what it is for an arbitrary statement to be given by some construction.

[22] This position is inconsistent with any position which takes there to be results which *conflict* with classical mathematics. A canonical example is the result that all functions on the real line are continuous [13, §3.3]. Our approach to constructive reasoning must reject all such counter-classical results. We are not alone in this—the constructivism of Bishop, Bridges and others agree on this point.

follow from the pre-theoretic notion. A construction proves $A \supset B$ if and only if when *combined with* any construction for A you have a construction for B. The assumption guiding Kripke models is that a construction for $A \supset B$ combined with one for A will be a *stronger* construction.[23] $A \supset B$ is true at c if and only if any stronger construction d for A is also a construction for B.

Constructions are *incomplete* and hence should not be expected to construct, for every claim A, either it or its negation $\sim A$. Constructions have computational content, so a construction of $A \vee B$ should be a construction of A or a construction of B. This jointly ensures that $A \vee \sim A$ ought fail. This cannot necessarily be constructed.

What is important, here, is that for a pluralist it does not follow that $A \vee \sim A$ is not true, or even, not *necessarily* true. It is consistent to maintain that all of the truths of classical logic hold, and that all of the arguments of classical logic are *valid* with the use of constructive mathematical reasoning, and the rejection of certain classical inferences. The crucial fact which makes this position consistent is the shift in context. Classical inferences are valid, *classically*; they are not *constructively* valid. If we use a classical inference step, say the inference from $\forall x(A \vee B)$ to $\exists x A \vee \forall x B$, then we have not (we think) moved from truth to falsity, and we cannot move from truth to falsity. It is impossible for $\forall x(A \vee B)$ to be true and for $\exists x A \vee \forall x B$ to be false; however, such an inference *can* take one from a truth which can be constructed to one which cannot, as we have seen. So, the inference, despite being classically valid, can be rejected on the grounds of non-constructivity.

This pluralist account of constructive inference is not a view that will be shared by constructivists who wholeheartedly reject the use of classical inference. However, it is a view which does justice of what constructive reasoning is. When a constructivist says 'not', she means *not*; she does not mean something else, foreign to the classical mathematician.[24] The constructivist differs from the classical reasoner only in her use of tighter canons of inference. It is hard to see how any other view can do justice to the practice of constructive mathematics. It seems that classical dogmatists must either reinterpret constructivist claims as being about something else (when she says $\sim A$ she means not that A is not true, but instead that A can be *proved* not to be true) or that intuitionistic logic merely a formalist game in which the rules are *syntactically* restricted to allow a more limited repertoire of proof.

VI. Criticism

We have given an account of logical pluralism, and we have shown how it contributes to our understanding of different traditions in contemporary Logic. In this section we address a few criticisms.

[23] This is the assumption challenged by relevant accounts of implication. In constructive mathematics, where relevance is not at issue, this account is appropriate.

[24] We use the example of the *mathematician* merely because constructive reasoning is most developed in this tradition. It need not be restricted to mathematics. Mathematical technique is *applied* when talking about the environment. We can reason constructively not only about the real line, but also about spatial and temporal distances, physical quantities, and many more things besides.

Anything Goes?

Objection: 'You say that there are many, many different consequence relations, and that none of these, in any objective, universal sense, is *better* than the others. Does it not follow that *anything goes*? On your view, there is no disagreement about logical consequence. But that makes a mockery of the current state of play in Logic. Stephen Read writes:

> Rival logical theories, such as intuitionistic logic, paraconsistent logics, relevant logics, connexive logics, and so on, are based on different philosophical analyses of this basic notion. [40, p. 36]

'According to your view, these logics are not *rivals*, they live in one large happy family.' Similarly, Graham Priest writes:

> Whether or not any of the nonstandard logics discussed here [intuitionist, many-valued and quantum, relevant and paraconsistent, conditional and free] are correct, their presence serves to remind us that logic is not a set of received truths but a discipline where competing theories concerning validity vie with each other. [37]

'On your account, such theories do not *compete*. You have misunderstood contemporary Logic.'

Reply: Pluralism is not a recipe for wholesale agreement. There can be disagreements about logical consequence. Our pluralism holds that *some* formal logics can fruitfully be seen as different elucidations of (V), the pretheoretic notion of logical consequence, and that (V) does not determine *one* logic, but rather, a number of them. It does not follow that there are no disagreements about notions of logical consequence. It does follow, however, that in any such disagreement the ground has to be fixed to ensure that the disputants are not talking past each other. To see this, consider the following two examples of genuine disagreement.

To begin, we disagree with intuitionists [13] who hold that there are arguments from A to $B \vee \sim B$ with a true premise and untrue conclusion. We disagree; we take every instance of $B \vee \sim B$ to be true. This disagreement, however, is entirely consistent with our pluralism.

Perhaps a more telling illustration arises within our own pluralistic ranks, and in particular on the issue of *dialetheism*, according to which contradictions may be true.[25] Dialetheists maintain that there are arguments of the form $A \vee B, \sim A \vdash B$ which are not only *invalid* but which have true premises and an untrue conclusion. Now, while both of us agree that the given argument is invalid—there are cases in which the premises are true and the conclusion untrue (viz., inconsistent situations)—we disagree *with each other* on the issue of whether the actual world is a case in which the premises are true. One of us (JC) endorses dialetheism; the other (Greg) does not. Still, despite this disagreement

[25] For discussion of dialetheism see the work of Graham Priest, who with Richard Sylvan coined the position. See Priest [35, 36].

within our own ranks neither of us has transgressed our pluralist commitments. The point of disagreement is a genuine one; however, it is an issue on which pluralism is neutral.

Accordingly, disagreement is possible; it is possible once we have set the terms of the debate. In both cases, with paraconsistent and intuitionist logic, we find a place for these non-classical logics—for both are elucidations of the pretheoretic notion (V) of logical consequence.

On the other hand, there might be other logics for which we can find *no* place in our catalogue of True Logics, as much as we admire their technical subtlety.[26] There are too many modal logics to hold each of them as the logic of broad metaphysical necessity. So, given a particular interpretation of each of the symbols in our formalism (including *consequence*) we can admit that there is a great deal of scope for rivalry. For the propositional modal logic of necessary truth preservation, a logic somewhere between S4 and S5 may be a candidate for getting things *right*. If so, then anything else *gets it wrong* when it comes to metaphysical necessity. There is scope for rivalry and disagreement when the meaning of the basic lexicon is settled. The moral of our pluralism goes as follows: Once you are specific about what your logic is meant to do, there is scope for genuine disagreement.

This raises a general question: What is it to *disagree* with an account of consequence? What kinds of disagreement are possible? There are at least four different ways in which disagreement and difference between formal logics can be understood. Here is a rough spectrum of what one might think about a logical system L.[27]

* *Abstract Geometries*: L is a *logic* because it is formally similar to other logics. It models a consequence relation. It is to logical systems what a finite projective geometry is to Euclidean geometries. Euclidean geometries and their close neighbours are used to model physical space. A finite projective plane is not going to be used to model physical space, but it may be used to model something analogous to physical space. Similarly, system L might be used to study something analogous to consequence relations. And so, it is called a logic for reasons of structural similarity.

* *Applied Geometries*: Take two geometries, a three-dimensional Euclidean space, and a particular non-Euclidean three-dimensional space. These two spaces might be *competing* models for the physical space in our region. Here the geometries are *applied*, for there is a notion of what it is to which the theoretical entities must correspond. Once rules of application of the model are settled, there is scope for a genuine *disagreement* between the two theories. Similarly, once applied, there is a scope for genuine disagreement between logical systems. However, this disagreement comes about *simply* by applying the logic to model the validity of real argument. Different formal systems can be equally appropriately used to model the validity of arguments. The analogy with applied geometry becomes appropriate only once the pretheoretic account (V) is fleshed out. Once you have a specific account of what kind of cases are in use (be they, worlds, constructions, situations) then there is scope for disagreement.

[26] Chief among those left out of our catalogue involve any systems for which transitivity or identity of consequence fails. For example, the Martin and Meyer system S-for-Syllogism, which rejects $A \vdash A$ on grounds of circularity, is ruled out given the lack of reflexivity [26, 30]. Moreover, Tennant's 'relevant logic' [51], which rejects transitivity, likewise fails to fall under the banner of logical consequence given in (V).

[27] Thanks to Daniel Nolan for discussion on this point.

* *Different Subject Matter*: We do not know how to label this position. *X* thinks what *Y* is doing is attempting to get at the same kind of thing as what *X* is trying to get at, but *Y* is going about it in completely the wrong way, and is actually either doing gibberish or talking about something else. The intuitionist view of the classicalist, or vice versa, *can* be seen like this, but need not be. A debate between the two which hinges upon whether the *proper* analysis of meanings ought proceed by way of truth conditions or in terms of provability or evidence conditions can be seen in this way.

* *Pluralism*: Finally, you can hold that two different logics *L* and *L'* are *both* accurate and systematic accounts of (different specialisations of) the one notion of logical consequence. We hold that this position is the appropriate one in each of the cases we have discussed.

All points on this spectrum are inhabited in debates between rival logics. Furthermore, we think that useful things can be said about the different ways in which plurality can arise. Pluralism comes in different *axes*. One is the difference between models and what is modelled. Logic can deal with both models (say, Tarski's) and what is modelled (say, possible worlds, or situations). You might have a preferred site on this axis, yet *still* allow a degree of plurality. For example, you might allow variance over the size of the domain of quantification (empty domains, proper classes), or you might allow plurality over the kinds of situations (or models) considered. These three kinds of pluralism are independent of each other. We have advanced each variety here, but one is enough to justify logical pluralism.

One True Logic After All?

Objection: 'Another potential problem with pluralism comes from the other direction. You have shown that there is a number of different ways that "case" can be interpreted in (V). But (V) has a *universal* quantifier in the front. (V) says that an argument is valid if and only if in *all* cases in which the premises are true, so is the conclusion. Is not *real* validity then preservation of truth across *all* cases? Will this not mean that the *true logic* is the intersection of all logical systems given by (V)? You have *one true logic* after all.'

Reply: Firstly, classical first order logic *is* logic after all. If the premises of a classically valid argument are true, so is the conclusion.[28] Those arguments are *valid*. They are not all constructively valid, or relevantly valid, but this does not stop them being valid, in an important and useful sense. The class of all Tarskian models is an important and natural class of cases, and it is appropriate to restrict our quantifiers in (V) to those cases.

Secondly, we see no place to *stop* the process of generalisation and broadening of accounts of cases. For all we know the only inference left in the intersection of all logics might be the *identity* inference $A \vdash A$. How bizarre it would be say that *identity* is the only valid argument. It seems a much more appropriate use of the term to call each of these systems *logic*.

Thirdly, each formal system is used to regulate inference, each falls under our original pretheoretical banner for logical inference. So, each of them are *logics*.

[28] This assumes that the actual world is consistent. Most readers will agree with this, as does one of the authors (Greg). We trust that enough has been said, however, to show that pluralism is neutral on the issue of whether the actual world is consistent.

Is This Conceptual Analysis?

Question: 'What is the status of your investigation? Are you engaging in a *conceptual analysis* of the concept of logical consequence?'

Reply: The nature of conceptual analysis is contested, so our remarks must be tentative. As many have noted, Tarski aimed to give an analysis of the 'intuitive' notion of consequence. Etchemendy repeats the story:

> Tarski begins his article by emphasizing the importance of the intuitive notion of consequence to the discipline of logic. He dryly notes that the introduction of this concept into the field 'was not a matter of arbitrary decision on the part of this or that investigator' (1956, p. 409). The point is that when we give a precise account of this notion, we are not arbitrarily defining a new concept whose properties we then set out to study—as we are when we introduce, say, the concept of a group, or that of a real closed field. It is for this reason that Tarski takes as his goal an account of consequence that remains faithful to the ordinary, intuitive concept from which we borrow the name. It is for this reason that the task becomes, in large part, one of conceptual analysis. [18, p. 2]

In so far as Tarski was doing conceptual analysis in his 'On The Concept of Logical Consequence', we are too.[29] We are not introducing a new concept and recommending that people study it. (V) captures the pretheoretic notion to which Tarski held his own account accountable. (V) is the most important guide to logical theory, and it does not constrain the field down to one candidate. Instead, it leaves the field open for a great deal of 'play'.

VII. Conclusion

Logic is a matter of truth preservation in all cases. Different *logics* are given by different explications of these cases. This account of the nature of logical consequence sheds light on debates about different logics. They arise from different accounts of the 'cases' in which claims are true or not. Once this realisation is made apparent disagreements between some formal logics are shown to be just that: merely *apparent.* A number of different formal logics, in particular, classical logics, relevant logics and intuitionistic logics, have their place in formalising and regulating inference. Each is an elucidation of our pretheoretic, intuitive notion of logical consequence. Such is our pluralism, which we have here tried to clarify. Two tasks remain: Showing that pluralism is superior to monism, and defending pluralism against objections. These are tasks we take up elsewhere [5].[30]

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[29] Tarski's own 'analysis' is captured more or less in (V). The apparent difference between him and us is that we, unlike him, take (V) to be neutral with respect to (V)'s cases.

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