A simple approach towards recapturing consistent theories in paraconsistent settings

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I believe that, for reasons elaborated elsewhere [5, 22, 23], the logic LP [3, 4, 19] is roughly right as far as logic goes.[1] But logic cannot go everywhere; we need to provide non-logical axioms to specify our (axiomatic) theories. This is uncontroversial, but it has also been the source of discomfort for LP-based theorists, particularly with respect to true mathematical theories which we take to be consistent. My example, throughout, is arithmetic; but the more general case is also considered.

1 Theories and logical closure

The problem, in short, arises as follows. Take the axioms of PA. Close under logic: namely, LP. Trouble: it is at least unclear whether the resulting theory is as strong as PA. What we want is that it *is* as strong as PA.[2] But without material modus ponens (or, equivalently, disjunctive syllogism) the resulting theory is likely not as strong. What we want to do is 'recapture' the consistent

[1] Familiarity with LP and its standard model theory is assumed. See [12, 24]. An appendix very briefly rehearses the 'semantics' for LP.

[2] I am assuming, throughout, that we have no (ultimately persuasive, etc) reason to think that the axioms of PA in fact describe an inconsistent phenomenon. This can, and has been, questioned [18, 23]; however, my point is a general one about 'recapturing consistent theories', and not so much about which theories are, in the end, truly consistent.

theory; we want to 'recapture' the consequences of LP-invalid rules such as (material) modus ponens and disjunctive syllogism.

2 One route: on the cheap?

One route is to be up front. In particular, those who take LP to be 'the one true logic' may nonetheless recognize limited roles for the classical closure operator. Specifically, one may specify one's theory by listing the PA axioms and invoking the classical operator (i.e., classical logic) for the closure role: one's theory of arithmetic is simply PA axioms closed under classical logic – and that's an end on it.

Nothing in this approach strikes me as philosophically inappropriate. Simply because one thinks that the true logic – the logic over the whole of one's language – is sub-classical doesn't mean that one thereby has foresworn all use of the classical closure operator. After all, closure operators are useful for specifying theories; and logic (i.e., true logic) may not always be the best suited (e.g., may be too weak) for closure over a proper theory in one's language. What one has foresworn, in championing LP as the one true logic, is simply that classical consequence is logic – that it is (let me say) truth-preserving across all sentences over all (relevant) points (etc.).

While there might not be anything philosophically improper with the invoke-classical-closure approach, different courses have in fact been pursued, including the addition of new logical vocabulary (e.g., new detachable conditionals) [20, 23, 29], or beefing up the logic by restricting attention to certain models [21, 23], or generalizing to multiple-conclusions framework and relying on extra-logical principles to recover the otherwise lost arithmetic truths [6, 7, 8, 13, 23]. While all such approaches may – probably do – have their (dis-) advantages, I turn to a simple idea, one that works even in detachment-free languages [10].

* * *Historical note on the target idea.* The target ('shrieking') idea, though independently discovered, is related to the approach in [6, 7] and, as Graham Priest conveyed in correspondence, finds direct roots in Priest's earlier work [23, 8.5]. Priest's approach is less economical than the suggestion here (e.g., Priest's approach applies to all formulæ full stop), and it is couched in a language with a detachable conditional. As I show below, the core idea is simpler and, importantly, applies even in 'detachment-free' languages, that is, languages with no modus-ponens-satisfying conditional [10].

The spirit of 'shrieking' (so to speak) also has the spirit – though not the letter – of the da Costa tradition of adding consistency operators [14]. One big difference (at least in letter) is that we are adding *rules* (non-logical rules) to specific theories; we are not adding new logical vocabulary – and, hence, elementary (logical) vocabulary of all theories in your language – in the tradition of da Costa's negation(s) operators. (And this can make a difference to target applications – e.g., truth theories, property theories, etc. In particular, adding new logical vocabulary can bring back more than you want: it can bring back, perhaps through the back door, the very same paradoxes and problems that motivated the initial drop to a weaker-than-classical logic. This is a well-known issue in applications of paraconsistent logic to glutty theories. See, for one example, the discussion of Curry's paradox and 'incoherent operators' in [5, Chs 2–3].)

Finally, the Asenjo–Tamburino logic [4], discussed in [11], also reflects a similarity in spirit: the idea there is to explicitly cordon off parts of the language as not susceptible to gluts (to truths with true negations). This approach, as with that of da Costa, winds up multiplying the logical vocabulary, rather than seeing the issue of 'essential non-gluttiness' as something theory-specific or extra-logical, as I see it in the shrieking method advanced in this paper. *End note.* * *

3 Shrieking theories: the basic idea

I focus on PA axioms, but the idea, with some qualifications, has applications to any axiomatic theory over domains we take to be 'non-glutty' (i.e., over domains whose true theories are negation-consistent) – almost all domains, by my lights [5], but I leave that debate for elsewhere.

3.1 Non-logical (shriek) *rules*

Unlike the case of (dual) paracomplete theories, there are no new *axioms* that we can add to the PA axioms that serve to 'recapture' target consistency. But axiomatic theories are a combination of axioms and rules that, jointly with the underlying logic (in our case, LP), make up the theory's closure operator. Because LP itself is too weak for target theories (say, PA), we want to add (non-logical) rules that, in effect, capture the target 'non-gluttiness' of the axioms while 'recapturing' the classical consequences. Throughout, I use $\vdash$

for LP consequence itself (see appendix for brief review) and $\vdash_{\mathrm{T}}$ for stating the proposed non-logical rules – or, in effect, for the given theory T's resulting closure operator (the given rules plus the background logic LP).

Let $\alpha!$, pronounced 'α shriek', abbreviate $\alpha \wedge \neg\alpha$.[3] The idea: arithmetic is simply achieved by taking the PA axioms and adding 'shriek rules'. Specifically, for each axiom α we add the $\alpha!$ rule:

$$\alpha! \vdash_{\mathrm{T}} \perp.$$

Such rules, together with the underlying logic (viz., LP), which governs all logical vocabulary occurring in such rules, make up the target closure operator for theory T (in the current case, PA). Such shriek rules are not, of course, *logical rules*; they're *non-logical*, theory-specific rules motivated by the (presumed-to-be-consistent) domain in question. As noted in §2, there's nothing – in principle – that bars adding whatever non-logical rules one pleases; but the shriek rules, attached to specific axioms (or more), enjoy a prima facie elegance that mightn't be shared by alternative choices of non-logical rules.

Some phenomena are 'glutty', in that their true theory is (negation-) inconsistent. Some phenomena are non-glutty – for example, arithmetical reality. What motivates the *shriek rules* is the desire to be up front, in the basic formulation of the axiomatic theory (the rules and axioms), that the target phenomena are non-glutty. In explosive logics (e.g., classical, intuitionistic) such shrieking is unnecessary; it's already going on 'silently' in the *logical* rules. In some theoretical contexts, perhaps only a proper subset of axioms are to be shrieked; but I concentrate here on the simplest case of a consistent theory.

3.2 $\mathbf{PA}_!$

Call an axiomatic theory *fully shrieked* just when all axioms enjoy shriek rules. A theory is shrieked (simpliciter) just if some axiom is shrieked. And so on. We concentrate here on fully shrieked PA, where the only (primitive) predicate is identity.

Let the fully shrieked PA theory (i.e., PA axioms closed under LP and shriek rules for all axioms) be $\mathrm{PA}_!$. In turn, an *LP model of PA*, and similarly

[3] '$\alpha!$' is also sometimes pronounced 'α bang', but the 'shriek' terminology sounds slightly more natural – and less aggressive.

LP models of PA$_!$, are defined standardly [18, 23].[4] Throughout, we use 'model' for 'LP model' (with the 'LP' implicit); and, given a model, P^+ and P^- are the extension and antiextension of predicate P, respectively.

Definition (Consistent models)**.** *Let D^n be the n-fold product of M's domain D. We say that a model M of theory T is* consistent *just if $P^+ \cap P^- = \emptyset$ for all predicates P in the language of T. A model of T is an inconsistent model of T iff $P^+ \cap P^- \neq \emptyset$ for some predicate P in the language of T.*

Definition (Consistent theories)**.** *A theory T is consistent just if negation-consistent; and T is inconsistent iff negation-inconsistent.*

Definition (Trivial models)**.** *Let D^n be the n-fold product of M's domain D. We say that a model M of theory T is* trivial *just if $P^+ \cap P^- = D^n$ for all predicates P in the language of T. A model of T is a non-trivial model of T iff it is not a trivial model of T.*

Definition (Trivial theories)**.** *We say that a theory T is* trivial *just if T contains all sentences of the language of T. We say that a theory T is* non-trivial *iff T is not trivial.*

Theorem 1. *Every non-trivial model of PA$_!$ is consistent.*

Proof. We show the contrapositive.[5]

Fact. *A model of PA$_!$ is inconsistent iff $P^+ \cap P^- \neq \emptyset$ for some predicate P in the language of PA.*

And since the only predicate in the language of PA$_!$ is identity, we have an immediate corollary:

Corollary. *A model M of PA$_!$ is inconsistent iff inconsistent wrt the identity predicate (viz., Id) iff for some terms t_1 and t_2 we have $M \models_{\mathsf{t}} Id(t_1, t_2)$ and $M \models_{\mathsf{t}} \neg Id(t_1, t_2)$, and so iff $M \models_{\mathsf{t}} Id(t_1, t_2)$ and $M \models_{\mathsf{f}} Id(t_1, t_2)$.*

Now, suppose that M is an inconsistent model of PA$_!$, and so $M \models_{\mathsf{t}} Id(t_1, t_2)$ and $M \models_{\mathsf{f}} Id(t_1, t_2)$ for some terms t_1 and t_2; and so $\langle \delta_M(t_1), \delta_M(t_2) \rangle \in Id^+$ and $\langle \delta_M(t_1), \delta_M(t_2) \rangle \in Id^-$. Since, by regularity of identity, $\delta_M(t_1) = \delta_M(t_2)$,

[4] NB: models treat identity as *regular*: $M \models_{\mathsf{t}} Id(t_i, t_k)$ iff $\delta_M(t_i) = \delta_M(t_k)$, where $\models_{\mathsf{t}}$ is the truth relation, δ denotation, and Id the identity predicate. (See the appendix for brief review.)

[5] I'm grateful to Greg Restall for discussion of this short proof idea in the case of PA$_!$.

we also get that $\langle \delta_M(t_1), \delta_M(t_1)\rangle \in Id^-$. This, together with addition axioms (or, similarly, multiplication axioms) delivers the result. In particular, axiom

$$\mathsf{Add}.\ \ \forall x\, Id(x+0,x)$$

implies that $M \models_{\mathsf{t}} Id(t_1+0,t_1)$. Regularity gives that $\delta_M(t_1+0) = \delta_M(t_1)$. But since $\langle \delta_M(t_1), \delta_M(t_1)\rangle \in Id^-$, we have that $\langle \delta_M(t_1+0), \delta_M(t_1)\rangle \in Id^-$, which implies that $M \models_{\mathsf{f}} Id(t_1+0,t_1)$. Hence, as one of Add's instances is false-in-M, we have that $M \models_{\mathsf{f}} \forall x\, Id(x+0,x)$, and so $M \models_{\mathsf{t}} \neg\forall x\, Id(x+0,x)$. But, now, Add's shriek rule delivers $\perp$. Triviality. □

Upshot. Since, almost by definition, every consistent LP model of $\mathrm{PA}_!$ is a classical model of $\mathrm{PA}_!$ (and vice versa), Thm 1 delivers:

Theorem 2. *Every non-trivial LP model of $PA_!$ is a classical model of $PA_!$.*

Are there classical models of $\mathrm{PA}_!$? Yes:

Fact. *M is a classical model of $PA_!$ iff M is classical model of PA.*

Proof. $\mathrm{PA}_!$ differs from PA only in 'adding' (as non-logical) the shriek rules; but such rules are already (logical) rules in classical PA. □

Hence, despite the invalidity of modus ponens and disjunctive syllogism (and more), LP-based glut theorists can 'recapture' consistent arithmetic via shrieking.[6]

4 General method: predicate shriek rules

What of the general method? Can we simply shriek all axioms of a theory and thereby 'recapture' the corresponding classical theory of the given domain?

Not surprisingly, the answer depends on the shape of theory. In the case of PA, we have exactly one predicate available, and hence exactly one avenue towards gluttiness: namely, glutty identity. Moreover, the shape and content

[6]It is worth noting that the trivial model $M_\perp$ can be added to the class of classical models without affecting classical logic. (Of course, if $M_\perp$ is added, it will be the unique inconsistent classical model.) On this approach, the foregoing results deliver that the models of $\mathrm{PA}_!$ are precisely the classical models.

of the PA axioms afford a simple 'classical recapture' via shrieking. Things mightn't always be so simple.[7] Still, the basic shrieking idea does generalize.

Let us suppose that we take a domain (or phenomenon) to be consistent; we take its true theory to be the sort of theory for which 'classical recapture' makes sense. Suppose that we aim to give the phenomenon an axiomatic theory. In taking the given domain to be consistent, we reject that the true axiomatic theory is inconsistent; we reject that there are predicates of the theory's language that deliver gluts. But how does our theory reflect this?

We can't add *axioms* to the theory that force it to be consistent; but we can add appropriate shriek rules. In particular, piggy-backing on [8], define a *predicate P's shriek rule* thus:

$$\exists x_1, \ldots, \exists x_n (Px_1, \ldots, x_n \wedge \neg Px_1, \ldots, x_n) \vdash_{\mathrm{T}} \perp$$

Shrieking *all* predicates in the language of one's theory suffices to ensure the analogue of Theorem 1: namely, that the only non-trivial models of the theory are consistent (indeed, classical) models.[8]

5 Philosophical question and reply

Why think that a given phenomenon – say, arithmetic – is in fact glut-free?[9] Why think that its theory should be fully shrieked? After all, once we have embraced a paraconsistent logic, are we not now open to the possibility of many gluts – many truths whose negations are also true?

Reply. My own view – though, I admit, perhaps not the view of some of the more famous or outspoken glut theorists [22, 23, 26, 27] – is that we know that the 'non-semantic world' (if you will) is glut-free, and as yet have no reason to doubt as much. As broad background epistemology, I subscribe to so-called epistemic conservatism in the spirit (though not letter) of Thomas Reid [25] and some of the contemporary pragmatists, including Harman [15]:[10] we are (at least prima facie) justified in maintaining what

[7] Correspondence with Greg Restall sharpened my thinking on this point. Zach Weber also raised a worry about how the method is to be generalized.

[8] Again, treating $M_{\perp}$ as a classical model removes qualifications about non-trivial models (though this is simply a terminological point).

[9] I'm grateful to an anonymous referee for prompting me to at least flag this issue. More can be said, but I simply give the issue and basic direction of my reply here.

[10] See too William Lycan's work [17], though this is a very minimal version of epistemic conservatism.

we accept and reject until we have some special reason to change. And with the vast majority of thinkers, I see no good reason to accept that the non-semantic realm (arithmetic, physics, etc.) might be hiding some metaphysical 'glutty' (contradictory) nature. The paradoxes, I maintain, give us special reason to drop to a subclassical logic; but they don't thereby give us reason to suspect that every domain, every phenomenon, is potentially glutty. The only inconsistencies are the bizarre but well-known semantic paradoxes, which are simply 'spandrels of truth' that have no significant metaphysical consequences [5].[11] Pending good reason to doubt as much, I maintain that our true theories of arithmetic – and theories involving non-semantic predicates generally – are to be properly shrieked.

6 Closing remarks

The *general shrieking method* involves three steps:

1. Set out one's axioms.
2. Add *non-logical* shriek rules – either shrieked-axiom rules or, more fundamentally, shrieked-predicate rules.
3. Close under the resulting closure operator: LP plus shriek rules.

Whether the target result is equivalent to the classical closure of one's axioms depends on the level of shrieking. If one shrieks all predicates of the theory (i.e., of the theory's language), one has the analogue of Theorem 1 for the theory. If one shrieks only the axioms (either some or all), one will, in general, achieve a stronger-than-LP theory that wears its consistency commitments on the sleeves of the theory.

With LP, as with other subclassical paraconsistent logics, our axiomatic theories don't show consistency commitments via new axioms; they show it via shriek rules, at least on one natural approach – as I hope to have shown.[12]

[11] The semantic version of Russell–Zermelo's paradox is also a spandrel of truth, though a spandrel of the predicate 'true of' (e.g., every predicate is true of some property which is exemplified by all and only the objects of which the predicate is true). I leave the 'spandrel of truth' account for another venue.

[12] Acknowledgements and updates. The idea in this paper emerged in conversation with Graham Priest on Wormwood Hill Road in Connecticut as we were discussing Nick Thomas' (unpublished) approach towards 'recapturing consistent theories' [28]. I'm grate-

Appendix

This appendix offers a very brief rehearsal of the 'semantics' of LP.[13] A fuller discussion is available in many places, for example [12, 24]. This presentation duplicates some of the presentation in [11], though adds a translation of the $\models_{\mathsf{t}}$ and $\models_{\mathsf{f}}$ relations used in the body of the current paper.

A First-order LP (with identity)

LP [3, 4, 19] is dual to K3 [16], both proper sublogics of classical logic (i.e., anything valid in such sublogics are valid in classical logic, though the converse fails). I focus on a common model-theoretic account of LP.

A.1 LP syntax

We assume a standard first-order syntax without identity (I discuss identity below in §A.5), taking $\forall$ and $\neg$ and $\vee$ as our primtive connectives (defining $\exists$ and $\wedge$ and $\supset$ in the usual way).

A.2 LP 'semantics'

An LP model M consists of a non-empty domain D, a denotation function δ, and a variable assignment v, such that:

- for any constant c, $\delta(c) \in D$,
- for any variable x, $v(x) \in D$,
- for any n-ary predicate P, $\delta(P) = \langle P^+, P^- \rangle$, where $\{P^+, P^-\} \subset \wp(D^n)$ such that $P^+ \cup P^- = D^n$. (We say that P^+ and P^- are the extension and antiextension of P, respectively.)

ful to Greg Restall for encouraging and very useful comments on a first draft, and to Nick Thomas who sketched proofs of equivalence of the fully shrieked PA system (viz., $PA_!$) and his 'congruence system' for PA. Thanks too to Dave Ripley and Zach Weber for comments, and to Michael Hughes for spotting infelicities in a late draft; and thanks very much to two anonymous referees for useful comments. Since the time of its writing, I have applied the ideas in this paper to various issues in the philosophy of logic and glut theory [7, 9].

[13] I am grateful to an anonymous referee for suggesting the inclusion of this appendix.

$|\varphi|_v$ is the semantic value of formula φ w.r.t. variable assignment v. This is defined recursively in familiar fashion, where $d(t)$ is $\delta(t)$ or $v(t)$, depending, as usual, on whether t is a constant or variable. For atomics:

$$|Pt_0, \ldots, t_n|_v = \begin{cases} 0 \text{ if } \langle d(t_0), \ldots, d(t_n)\rangle \notin P^+ \text{ and } \langle d(t_0), \ldots, d(t_n)\rangle \in P^- \\ 1 \text{ if } \langle d(t_0), \ldots, d(t_n)\rangle \in P^+ \text{ and } \langle d(t_0), \ldots, d(t_n)\rangle \notin P^- \\ \frac{1}{2} \text{ otherwise.} \end{cases}$$

The inductive clauses are as follows:

1. $|\varphi \vee \psi|_v = \max\{|\varphi|_v, |\psi|_v\}$.
2. $|\neg\varphi|_v = 1 - |\varphi|_v$.
3. $|\forall x \varphi|_v = \min\{|\varphi|_{v'} : v' \text{ is an } x\text{-variant of } v\}$.

Conjunction and existential quantification can be defined from these in the normal way.

A.3 Truth and falsity relations

Other notation, used in §3.2, is available for *truth in a model* and *falsity in a model*. These simply amount to satisfaction and satisfaction-of-negation. In particular, we define the following.[14]

Definition (Truth in a model)**.** *Let A be a sentence. Then $M \models_{\mathsf{t}} A$ iff $|A|_v \in \{1, .5\}$ for all variable assignments v.*

Definition (False in a model)**.** *Let A be a sentence. Then $M \models_{\mathsf{f}} A$ iff $|A|_v \in \{.5, 0\}$ for all variable assignments v.*

We say that a sentence A is *true in a model* M just when $M \models_{\mathsf{t}} A$, and *false in a model* M just when $M \models_{\mathsf{f}} A$. In LP (and similar paraconsistent logics), we can have some sentence A and model M such that A is 'glutty' (both true and false) with respect to the model: $M \models_{\mathsf{t}} A$ and $M \models_{\mathsf{f}} A$. A simple example is an atomic Pc such that $\delta_M(c) \in P^+ \cap P^-$.

[14] I give this notation only because I use it in the main body of the paper; it does not add anything to the foregoing semantics.

A.4 LP validity/consequence

We restrict the consequence (or validity) relation to sentences.[15]

Definition (LP consequence)**.** *Let A be a sentence, and X a set of sentences. $X \vdash A$ iff there's no LP model in which everything in X is true and yet A is not true, that is, no model M such that $M \models_{\mathsf{t}} B$ for all sentences B in X but $M \not\models_{\mathsf{t}} A$.*

LP consequence is paraconsistent: $A, \neg A \nvdash B$. A counterexample is suggested in §A.3. LP is not 'paracomplete', that is, excluded middle holds: $B \vdash A \vee \neg A$ (proof: exercise).[16]

A.5 Adding identity

We augment the standard syntax with a unary (identity) predicate Id. In LP, Id is treated as *regular*, which means that we constrain our models – what counts as an LP model – to those that treat identity statements $Id(t_1, t_2)$ as true just when the given objects are truly identical:

$$M \models_{\mathsf{t}} Id(t_1, t_2) \text{ iff } \langle t_1, t_2 \rangle \in Id^+ \text{ iff } \delta(t_1) = \delta(t_2).$$

The difference between identity in LP and identity in classical (and other standard) settings is that we can have 'glutty' identity claims: both $Id(t_1, t_2)$ and $\neg Id(t_1, t_2)$ being true in a model.[17] In other words, while the regularity of identity demands that identity claims be *true* iff the given pairs of objects are identical, such identity claims can also be false – as an independent matter. (I'm not arguing for this; I am simply presenting the treatment of identity relevant to the discussion in the paper.) In particular, we allow the

[15] There is no need, in principle, to do this; but it simplifies presentation. See any of the cited sources for a fuller account.

[16] As discussed above, LP and K3 are strict duals, which comes up very nicely in the multiple-conclusion versions LP$^+$ and K3$^+$, where, e.g., each logic enjoys exactly one of the following 'dual' patterns: $A, \neg A \vdash B$ and $B \vdash A, \neg A$. Discussion of LP$^+$, with some discussion of K3$^+$, may be found in [8].

[17] NB: Strictly speaking, one can go different ways on this. As usual, adding identity into the mix can be controversial, even among logicians who agree on the underlying (identity-free) logic. But I skip these issues here, and give only a standard account sufficient for understanding the 'shrieking' method.

antiextension of Id to be free of constraints and simply retain the standard falsity clauses:

$$M \models_{\mathsf{f}} Id(t_1, t_2) \text{ iff } \langle \delta(t_1), \delta(t_2) \rangle \in Id^-.$$

Accordingly, a pair of objects can be in the antiextension of Id even if also in the extension of Id, or it may be treated 'classically' by some models – a pair in exactly one of the extension and antiextension. This raises a notable point about classical models.

A.6 LP models and classical models

What should be plain is that LP models properly include all classical models. In particular, the only difference between classical models and LP models is that the former obey an exclusion condition on all predicates P, namely: $P^+ \cap P^- = \emptyset$. LP drops the exclusion clause; otherwise, LP models are exactly in line with classical models. In short: whatever counts as a classical model counts as an LP model; it's just that there are more things that count as LP models – namely, those otherwise classical models that transgress the exclusion condition for some predicate or other.

A.7 A note on FDE

Finally, it is worth noting that another prominent (proper) subclassical, paraconsistent logic is FDE [1, 2], for 'first-degree entailment' or, sometimes, 'logic of tautological entailments'.[18] This logic is a sort of combination of K3 and LP, being weaker than each one. Formally, one drops the requirement on LP models that $P^+ \cup P^- = D^n$ for each n-ary predicate, allowing some predicates to have empty extensions and antiextensions.

A salient difference between FDE and LP is that the latter enjoys excluded middle while the former, like K3, does not. (In increasingly standard jargon, both are paraconsistent but only FDE is paracomplete, since excluded middle holds in LP. Intuitively, paraconsistent logics tolerate negation-inconsistency while paracomplete logics tolerate negation-incompleteness.) Worth noting, however, is that the shrieking idea applies just as well to FDE, though to 'recapture' a fully classical theory one needs to go beyond

[18] I am grateful to an anonymous referee for prompting comments on FDE's relation to LP and shrieking.

shrieking; one needs to supplement the closure operator with (non-logical) rules/axioms that ‘bring back’ excluded middle, etc.

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