The simple liar without bivalence?

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1. Background

The *Simple Liar* is a sentence that says of itself only that it is false. The *Strengthened Liar* is a sentence that says of itself only that it is not true.

How does the Strengthened Liar differ from its (allegedly) weaker brother, namely, the Simple Liar?

According to standard thinking the Simple Liar, unlike the Strengthened Liar, yields inconsistency only if the following principles are assumed:

> Bivalence: Every sentence is either true or false.
> Non-Contradiction (LNC): No sentence is both true and false.

From the so-called truth principle, namely,

> Truth Principle (T): A sentence is true iff what it says is the case

one confronts inconsistency if, in addition to the existence of the Simple Liar, both (LNC) and Bivalence are assumed. Let *s* be the Simple Liar. Given

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that *s* says only that *s* is false we immediately get the following instance of (T):

(A) *s* is true if and only if *s* is false.

Given (LNC) and Bivalence, (A) yields inconsistency: either *s* is true or *s* is false (Bivalence). Given (A), either disjunct yields *s* is both true and false, which, given (LNC), is inconsistent.

The question is: does the existence of the Simple Liar generate inconsistency *without* Bivalence? According to the standard story the answer is 'no'. Indeed, that the standard answer is 'no' is precisely why the Strengthened Liar is so called. Bas van Fraassen (1968) coined the term 'strengthened liar'. As van Fraassen put it, '[the Strengthened Liar] was designed especially for those enlightened philosophers who are not taken in by Bivalence' (1968: 147), the implicature being that the weak, Simple Liar is sufficiently dismantled by a rejection of Bivalence.

Robert Martin (1984: 1–3), with support from Nathan Salmon, has challenged the standard story.[1] Martin argues that, *pace* standard thinking, the difference between the Simple and Strengthened Liars does not rest with Bivalence; the Simple Liar generates inconsistency without Bivalence.[2]

We find Martin's argument to be very interesting and, if sound, very surprising. As far as we know, no one has challenged the argument. Our aim in this paper is to take up the challenge. Specifically, we attempt to show that if, as Martin argues, the Simple Liar does generate inconsistency without Bivalence, then Martin's argument doesn't show as much, at least not if classical propositional logic (inference rules) may be assumed.

2. *Martin's argument*

Where by 'ordinary Liar' is meant *Simple Liar* (as above, §1) Martin writes:

> Since this way of distinguishing between the ordinary Liar and the Strengthened Liar has become fairly standard [viz., that the former but not the latter presupposes Bivalence], it is interesting to note that the ordinary Liar is actually just as independent of the principle of bivalence as its big brother. … In particular we can show, without any use

[1] Martin attributes the challenge, and the corresponding argument, to Nathan Salmon. We are inclined to call the relevant argument the *Martin-Salmon Argument*; however, only Martin is an explicit author of the published argument, and so we refrain from so dubbing the argument.

[2] Of course, the two paradoxes still remain distinct, inasmuch as different principles are used to generate the respective inconsistencies. Martin notes as much. His point is merely that the standard story, which takes Bivalence to be the distinguishing difference, is incorrect.

of the principle of bivalence, that the ... [existence of the Simple Liar] is also incompatible with (T). The argument ... does rely on some other semantic principles, besides (T); but no appeal is made to the principle of bivalence. Here is the argument:

> Let s_0 be the ordinary Liar. First, we show that s_0 is not false, as follows: suppose s_0 is false; then, since that is what it says, it is true, and hence not false. (Principle: no sentence is both true and false.) Therefore, s_0 is not false. But now we can see that s_0 is false, since s_0 says something the negation of which (s_0 is not false) is true. (Principle: a sentence is false if its negation is true.) Thus a contradiction. (1984: 2)

Before explaining why his argument fails we should first lay out the principles Martin explicitly invokes. In addition to (LNC) and (T) Martin explicitly invokes the following principle, which we shall dub Falsity:

FALSITY (F): If the negation of a sentence, $\langle A\rangle$, is true, then $\langle A\rangle$ is false.

For the sake of clarity we shall reformuate (F) and (T) as follows, where $\rightarrow$ is a conditional and $\leftrightarrow$ the corresponding biconditional:

(T*): $T\langle A\rangle \leftrightarrow A$
(F*): $T\langle \neg A\rangle \rightarrow F\langle A\rangle$

Note that (T*) is an acceptable representation of (T) given that we are concerned only with sentences for which the naive *says-that* principle is at work, according to which $\langle A\rangle$ says (only) that A. According to (T) a sentence is true iff what it says is the case. So, in particular, 'Tarski is on the mat' says only that Tarski is on the mat, and so is true iff Tarski is on the mat. This is the upshot of (T*).

3. *Why the argument fails*

The problem with Martin's argument is that (T*) and (F*), given classical propositional logic (CPL), *entail* Bivalence. That this is a problem is clear: The reason (F*) is formulated merely as a conditional, rather than a biconditional, is to leave open the possibility of so-called *gaps*, sentences that are neither true nor false (counterexamples to Bivalence). In particular, the possibility that the Simple Liar is gappy must be left open. And, indeed, this is the chief burden of Martin's argument: to show that, just like its strengthened brother, the Simple Liar generates inconsistency even if it itself is gappy. As the following shows, however, such a possibility is ruled out given CPL and Martin's assumed principles – namely, (T*) and (F*).

That (T*) and (F*) entail Bivalence may be seen as follows:

(0) $T\langle A\rangle \leftrightarrow A$ [T*]
(1) $\neg T\langle A\rangle \leftrightarrow \neg A$ [0, MTT]

(2)	$T\langle\neg A\rangle \leftrightarrow \neg A$	[T*]
(3)	$\neg T\langle A\rangle \leftrightarrow T\langle\neg A\rangle$	[1, 2, Transitivity]
(4)	$\neg T\langle A\rangle$	[Premiss, for CP]
(5)	$T\langle\neg A\rangle$	[3, 4, MPP]
(6)	$T\langle\neg A\rangle \rightarrow F\langle A\rangle$	[F*]
(7)	$F\langle A\rangle$	[5, 6, MPP]
(8)	$\neg T\langle A\rangle \rightarrow F\langle A\rangle$	[4, 7, CP]
(9)	$T\langle A\rangle \vee F\langle A\rangle$	[8, CPL equivalence and DNE]

As $\langle A\rangle$, above, is arbitrary we conclude that, given CPL, Martin's (T*) and (F*) entail Bivalence: that every sentence is either true or false, which is the import of (9).[3]

As Graham Priest pointed out (in conversation) there is another way to see the point: Martin's argument employs the principle of inference $A \rightarrow \neg A \vdash \neg A$, which is used essentially in the first stage of Martin's argument. This principle fails in standard truth-gap theories, including, for example, L_3.[4]

4. *Summary*

The upshot of our argument is that one cannot consistently allow that the Simple Liar is gappy if one also accepts (T*) and (F*); given classical propositional logic, these principles entail Bivalence and, so, a fortiori the principles entail that the Simple Liar is either true or false (and, hence, not gappy).

To be sure, Martin's argument does not *explicitly* invoke Bivalence, and even this much, perhaps, is interesting. Nonetheless, Martin's argument is philosophically or logically significant only if it shows, as Martin claims of it, that the Simple Liar is 'just as independent of the principle of Bivalence as its big brother'. The upshot of our argument is that Martin's 'demonstration' shows no such result, at least given classical propositional logic.

What remains an open question is whether Bivalence follows from (T*) and (F*) in other non-classical, and in particular 'gappy', logics – for example, intuitionistic logic. We hope to settle this question elsewhere. For

[3] Some might say that since Excluded Middle (LEM) holds in CPL Bivalence also thereby holds. This is incorrect. In order to get Bivalence from LEM and CPL one needs to add some principle(s) governing truth and falsity. What is surprising is that in addition to (T*) only the weak (F*) is required to yield Bivalence.

[4] L_3 is the same as K_3 (i.e., so-called *strong* Kleene) except that $A \rightarrow B$, while still definable in terms of $\neg$ and $\vee$, has the value *1* when A and B are 'gappy' (i.e., have the value i). (Hence, as only *1* is designated in L_3, a countermodel to the given principle arises when A is gappy, in which case $A \rightarrow \neg A$ is designated but $\neg A$, undesignated.) The principle doesn't fail in intuitionism. Whether our argument will apply to intuitionism remains open.

now, we conclude that Martin's 'demonstration' does not establish that the Simple Liar generates inconsistency without Bivalence, at least not if classical propositional logic may be assumed.[5]

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