Trivializing sentences and the promise of semantic completeness

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1 Background and aim

One of the best-known and intuitively gripping arguments for a glutty (i.e., negation-inconsistent) theory of language arises from the promise of semantic completeness. The thought, advanced at least in the 1970s by Routley [13] and Priest [9, 10], is that the full truth of our language may be achieved at a small price: namely, gluts – sentences such that the (familiar logical, 'extensional') conjunction of them and their (logical) negation is true according to the given theory (and so true if the theory is true). The promise is that despite the familiar paradoxes we may enjoy an English theory of English which is complete with respect to all truths about English.[1]

The argument from semantic completeness (or semantic closure) to glut theory runs via a paradox-driven *limitative disjunction*. In particular, familiar paradoxes in English deliver the familiar limitative disjunction of an incomplete theory and a negation-inconsistent (glutty) theory (of English in English). (I review the basic argument in §3.) According to the target thinking, this limitative disjunction is true; the choice is between incompleteness and (negation-)

[1]Historical note: Emerging at a time in philosophy when glut theory was without serious argument and at best pooh-poohed as shallow, the historical significance of the target argument from semantic completeness to gluts is large. In the mid-1950s and 1960s (and mid-70s), the contemporary pioneer of glut theory, namely, (mathematician) Florencio Asenjo [1, 2], took it as obvious that gluts are an important discovery from so-called paradox (or antinomies, as he called them); and he claimed, though *didn't argue* (as far as I can see), that we should not only accept them but learn from them, etc. What philosophers Routley [13, 17] and Priest [8, 9, 10] did, in addition to giving much clearer articulation of glut theory, was provide the necessary philosophical arguments for glut theory and engage in a wide-reaching campaign of applying glut theory (or 'strong paraconsistency') well beyond standard paradoxes. It was Priest and Routley (not without important help from Brady, Meyer, Mortensen, and other Australasian philosophical logicians) who articulated, championed, and defended the best-known reason for glut theory: namely, the argument from semantic completeness. And it is this now-rightly well-known sort of argument with which I am concerned in this paper.

inconsistency. And on the target thinking, incompleteness should be rejected in favor of inconsistency. As Routley (Sylvan) puts it:

> The liberating effect of giving up the classical faith – with its commitment to an underlying classical logic, to classical theories and in consequence to classical limitative theorems and to classical foundations for mathematics and linguistics – is immense: not only do many 'paradoxes' generating spurious philosophical problems in a wide range of intensional areas – such as meaning, analysis, information, evidence, confirmation, obligation, to list a few – vanish...; the logical and semantical antinomies cease to be paradoxical and become, what they always seemed to be, proofs; and the limitative theorems of Gödel, Tarski, and others [taken to establish unavoidably *incomplete* theories] lose their apparent universality and cease to apply [as arguments for incompleteness]. Therewith one is free to return to something like the grand simplicity of naive set theory, to semantically closed natural languages (having abandoned the towering but ill-constructed and mostly unfinished hierarchies of formal languages), and to intuitive accounts of truth, of proof...and of many other intensional notions. [13, p. 3]

And similarly Priest:

> The ad hocery, Gothic hierarchies and loss of expressive power required by the rejection of semantical closure could seem reasonable only to a logical community living, as Wittgenstein put it [Remarks on the Foundations of Mathematics], in superstitious fear and awe of contradiction. The time has come to put the superstition aside. [9, p. 128]

According to such thinking, the goal of a semantically complete English theory of English is paramount; and glut theory promises achievement of the goal.

I am a glut theorist; I hold that some truth is also false, that is, that the logical negation of the given truth is also true. But I reject the argument from the standard paradox-backed limitative disjunction to gluts. This paper gives my reason for rejecting such arguments. The main problem concerns directly analogous limitative disjunctions invoking *triviality* or *Post-inconsistency* instead of negation-inconsistency – 'absolute inconsistency' and not merely 'simple inconsistency', as some target writers put it [13]. The main problem arises with now-familiar so-called v-curry phenomena [3, 4, 5, 6, 7, 11, 14, 15, 18, 19, 20, 21], and in particular what I take to be the generalized version of it.[2]

[2]Some writers in the area of v-curry phenomena, from Beall [3] to Cook [5] to Murzi [7] to L. Shapiro [14] to Whittle [20] and others, have had the target application (to such arguments) in mind, to varying degrees of explicitness. My aim is simply to frame a simple version of the phenomenon directly against the target argument from limitative disjunction to gluts. The paper is an invitation to defenders/advocates of the semantic-completeness route towards gluts to explain, in simple and plausible terms, why the simple considerations here fail to undermine the target argument.

The structure of the paper is as follows. After a brief review of the relationship between paradoxes, theories, and closure (or 'delivery') relations, I review the familiar paradox-driven ideas and argument involved in the target argument from semantic completeness to gluts. That argument turns on a limitative disjunction involving incompleteness and negation-inconsistency (gluttiness). In turn, I present a directly analogous argument for another limitative disjunction, one involving incompleteness and Post-inconsistency (triviaility). Parity of reason demands equal treatment of both limitative disjunctions (and conclusions drawn therefrom). But, then, we should choose incompleteness. I close with a very brief remark on whether any paradox-backed arguments for gluts remain.

2 Paradoxes as apparently trivializing sentences

Among a language's sentences may be genuinely paradoxical sentences. Such sentences are not problematic in virtue of living in the language; they pose problems for *theories* in the language.

Theories are simply sets of sentences. And there is nothing difficult about having a theory that expresses typical paradoxical sentences: the boring singleton theory containing (say) a liar or a liar-instance of suitable T-biconditionals is indeed boring but also doesn't deliver absurdity.

Delivery of absurdity requirers a delivery relation. And that's where genuine paradox comes alive. If we close the otherwise boring theory containing familiar liar-like sentences under all necessary-truth-preserving rules (including, for example, rules governing 'true'), we have an immediate brush with absurdity: we appear to be left with the so-called trivial theory, which is simply the theory containing all sentences of the language of the theory. That theory is the uncontroversial bedrock of absolute badness.

It is *closed theories* – that is, theories closed under the theory's absence-of-counterexample delivery relation – that paradoxical sentences challenge. Indeed, in the end, it is closed theories in terms of which genuinely paradoxical sentences are genuinely paradoxical. Such sentences appear to trivialize our given theories.

'Solutions' to paradoxes explain why the apparently trivializing sentences aren't in fact trivializers for the given theory. Such solutions typically take one of two routes: point to expressive limitations of the language of the given theory, or point to the closure relation for the given theory. In the former case (viz., language limitations), one generally says that the apparent trivializers are not in fact sentences of the language of the theory. In the latter case (viz., features of the closure relation), one points either to logic itself (i.e., the base or universal closure relation for all of our theories) or to the extra-logical, theory-specific ingredients (e.g., rules) of the given closure relation; in either direction, one explains that the apparent delivery of the trivial theory gets arrested somewhere somehow. Either way, be it via language limitations or closure-relation features, the apparent trivializers are either not in our closed (true) theories or they are but in fact are harmless curiosities – not trivializers after all.

Such observations form the background of the best-known argument for a glutty theory of (at least) English, the argument from semantic completeness.

3 Limitative disjunction one

Glut theorists who endorse the argument from semantic completeness to gluts endorse the argument from familiar paradox (e.g., liar, 'grounding', etc.) to the central limitative disjunction. I review the familiar idea and argument here, focusing on liar-paradoxical phenomena and truth.

3.1 Adequate truth predicate

Let $\mathcal{T}$ be a theory (a set of sentences) in language $L_{\mathcal{T}}$, closed under the delivery – that is, entailment or absence-of-relevant-counterexample – relation $\vdash_{\mathcal{T}}$.

A truth predicate T is *adequate* with respect to $\langle \mathcal{T}, \vdash_{\mathcal{T}} \rangle$ if and only if for all sentences A in $L_{\mathcal{T}}$, the theory's delivery relation $\vdash_{\mathcal{T}}$ delivers A from $T\langle A \rangle$ and vice versa, where $\langle A \rangle$ is a name of A.[3]

Assuming (throughout) that any relevant delivery relation is an entailment – or *absence of relevant counterexamples* – relation, we may say that T is adequate with respect to $\langle \mathcal{T}, \vdash_{\mathcal{T}} \rangle$ just if there is no relevant counterexample to the arguments $A \therefore T\langle A \rangle$ and $T\langle A \rangle \therefore A$.[4]

3.2 Limitative disjunction one: incomplete or glutty

Claim: any true English theory of English is either incomplete with respect to all truths about English or simply (i.e., negation-) inconsistent – that is, either incomplete or glutty.

Proof: Let 'true' be adequate with respect to $\langle \mathcal{T}, \vdash_{\mathcal{T}} \rangle$, where $\mathcal{T}$ is our would-be complete but consistent (English) theory of English closed under the delivery relation $\vdash_{\mathcal{T}}$. Now consider:

c. (c) is not true.

Assuming that logic (i.e., the base, universal, topic-neutral delivery relation on all of our theories) imposes excluded middle, we have that (c) is true or not. Given that 'true' is adequate with respect to $\langle \mathcal{T}, \vdash_{\mathcal{T}} \rangle$, we get that (c) is true and not – and so a glut. Hence, if $\mathcal{T}$ is complete, $\mathcal{T}$ is glutty; and so follows the limitative disjunction: any true English theory of English is either incomplete or simply (i.e., negation-) inconsistent – either incomplete or glutty.

[3] One also imposes adequacy conditions for a truth predicate's interaction with logical negation (which, note well, need not follow directly from the simple capture-release rules for the truth predicate); but I ignore this here, as the issues are well-known.

[4] As per the previous footnote, adequacy generally also involves no counterexamples to corresponding arguments involving other logical connectives; and, again, this needn't follow directly from the simple adequacy condition involving only truth.

4 From limitative disjunction to gluts

There are various well-known responses to this paradox-driven argument for the given limitative disjunction. My concern is with the target glut-theoretic response. Here, the response is that there is a truth predicate in English which is adequate with respect to any true theory of English; but the complete such theory is (negation-) inconsistent; the delivery relation has a weak base relation (i.e., logic); in particular, the 'dual' of excluded middle – namely, 'ex falso quodlibet' or 'ex contradictione quodlibet' – is not forced by the theory's delivery relation.

The glut-theoretic argument treats semantic completeness – the full truth of English in English – as a prime goal. This goal, together with the (first) limitative disjunction, drives us to gluts. Such is the thought.

5 Limitative disjunction two

The problem with the argument from semantic completeness to gluts is a perfectly parallel limitative disjunction of incompleteness and absolute inconsistency (triviality, Post-inconsistency). As the cases are exactly parallel, parity of reason demands parallel treatment; and hence we're to accept a perfectly parallel argument away from semantic completeness to absolute inconsistency – absurd. The problem arises from a particular version of so-called v-curry phenomena – what I dub *the trivializer 'paradox'*.

5.1 Adequate trivializer predicate

A *trivializing sentence* is a sentence that, relative to a theory's delivery (or closure) relation, delivers the trivial theory – the theory containing all sentences of the language of the theory. Letting $\bot$ be some 'explosive' sentence (e.g., in many theories, 'everything is true'), we say that A is a $\langle \mathcal{T}, \vdash_{\mathcal{T}} \rangle$-*trivializer* if and only if A delivers $\bot$ according to the given delivery relation, that is, in symbols, just if $A \vdash_{\mathcal{T}} \bot$. The existence of such sentences, at least in the languages of target semantically complete theories (e.g., English), is not in question.

A *trivializer predicate for a theory (and delivery relation)* is a predicate true of all and only the given trivializers. While there are (at least) two different adequacy conditions that partially characterize two different uses of 'trivializer predicate', my focus is on what I shall call *narrow adequacy*.[5]

A trivializer predicate is *narrowly adequate* with respect to a theory and its delivery relation just if for any sentence A of the theory's language the truth

[5] I'm grateful to Lionel Shapiro who (in conversation) pushed me to be explicit about my focus on narrowly adequate trivializer predicates (see below). For readers unfamiliar with the flurry of so-called contraction-free approaches to paradox [3, 4, 5, 6, 7, 11, 14, 15, 18, 19, 20, 21], I note that such approaches to paradox make very strong distinctions among ways of combining sentences (recognized by the theory's delivery relation); but target semantic-completeness theorists recognize that one such way is the standard, familiar (extensional) conjunction from standard logic classes – and this is what's involved in narrow adequacy.

of A extensionally (materially, in-fact, etc.) conjoined with $Triv\langle A\rangle$ delivers, according to the given delivery relation, the trivial theory.

In symbols: let $\wedge$ be (so-called extensional) logical conjunction. (This is familiar, standard logical conjunction.) Then a trivializer predicate $Triv$ is *narrowly adequate* with respect to $\langle \mathcal{T}, \vdash_{\mathcal{T}} \rangle$ if and only if

$$A \wedge Triv\langle A\rangle \vdash_{\mathcal{T}} \bot$$

where $\bot$ itself delivers, according to $\vdash_{\mathcal{T}}$, all sentences of L_T.

The problem for the glut-theoretic argument from semantic completeness arises from narrowly adequate trivializer predicates.

5.2 Limitative disjunction two: incomplete or trivial

Claim: any true English theory of English is either incomplete with respect to all truths about English or absolutely (i.e., Post-) inconsistent – that is, either incomplete or trivial.

Proof: Let 'trivializer' be narrowly adequate with respect to $\langle \mathcal{T}, \vdash_{\mathcal{T}} \rangle$, where $\mathcal{T}$ is our would-be complete but non-trivial (English) theory of English closed under the delivery relation $\vdash_{\mathcal{T}}$. Now consider:

d. (d) is a $\langle \mathcal{T}, \vdash_{\mathcal{T}} \rangle$-trivializer.

For convenience, let $Triv$ be the given (narrowly adequate) trivializer predicate, so that (d) has the form (indeed, just is the sentence) $Triv(d)$.

Since 'trivializer' is narrowly adequate with respect to $\langle T, \vdash_T \rangle$, we have, by substitution of identities, that $Triv(d) \wedge Triv(d)$ delivers $\bot$, and so delivers the trivial theory. But, then, any relevant possibility – recognized by the given theory's delivery relation – in which $Triv(d) \wedge Triv(d)$ is true is one in which the trivial theory is true. But $Triv(d) \wedge Triv(d)$ is equivalent, according to the logical base of the delivery relation (i.e., according to logic itself), to $Triv(d)$. Hence, there's no relevant possibility in which (d) is true but the trivial theory (in English) untrue. This is precisely the necessary and sufficient mark of any relevant $\langle T, \vdash_T \rangle$-trivializer; and so (d) itself is true. Hence, the limitative disjunction: any true English theory of English is either incomplete or absolutely (i.e., Post-) inconsistent – either incomplete or trivial.[6]

6 From limitative disjunction to triviality

The target argument for glut theory crucially invokes the negation-inconsistent version of the limitative disjunction; and that disjunction is backed by familiar 'paradoxical' argument (e.g., from liar-like phenomena or the like). Since, as advocates of the target argument emphasize, the limitative disjunction and the paradox-based argument(s) behind it are carried out in English, we cannot find

[6] For experts on so-called v-curry: the last steps in this trivializer paradox exhibit a *strengthened* pattern, as Lionel Shapiro dubs it [15].

fault with the language of such would-be complete theories. (We can't reject that the target notions are expressed in English.) Instead, the thought is that the full truth about English is a negation-inconsistent theory.

The problem: what the absolute-inconsistency (or triviality) version of the limitative disjunction shows is that, by parity of reasoning, the full truth about English is trivial. That's absurd if anything is.

We should reject the target argument from the negation-inconsistency version of the limitative disjunction and the goal of completeness to gluts. Even if, contrary to appearances, parity of reason doesn't demand equal response to the perfectly parallel limitative disjunctions, the absolute-inconsistency version of the limitative disjunction puts paid to any promise of a complete English theory of English; and so the promise of having such a theory at the mere price of gluts is a false promise.

7 Objections and Replies

Objection 1. The given trivializer paradox is a generalized version of so-called v-curry phenomena, generalized to apply to the theoretically important notion of *trivializing sentences* and any *absence-of-relevant-counterexample* delivery relations on theories – and not merely the base (universal, etc.) closure relation *logic* or *logical entailment*. But v-curry phenomena of any stripe are to be dealt with by rejecting the existence of *any* so-called contracting closure (or, generally, delivery) relations for our true theories, where contracting relations sanction 'contracting' an acceptable combination of a sentence and its equivalent into the sentence itself.

Reply 1. The objection is offline. No contraction of sentences is involved in the (note well: narrow-adequacy) trivializer argument – certainly none that isn't sanctioned by the lights of glut theorists behind semantic-closure arguments.

Objection 2. We should reject the use of 'trivializing sentence' that allegedly corresponds to the given narrow-adequacy condition. While there may be no explicit contraction of sentences involved in the trivializer argument (behind the second limitative disjunction), there is *implicit* contraction going on in the narrow-adequacy condition. In particular, the only way to justify the narrow-adequacy condition for trivializer predicates is to use some form of contraction for the target delivery relation.

Reply 2. The justification for narrowly adequate trivializer predicates arises from reflection on delivery relations and trivializing sentences.

A delivery relation $\vdash_T$ delivers the consequences of whatever's true according to the theory T. If we have that A is true according to theory T, *and* – the simple, extensional logical 'and' – we have that A delivers B according to $\vdash_T$, then we have that B is true according to T too – that is, that B by itself is also among the truths (according to T). That's just what delivery relations do. We aren't *proving* this about delivery relations; we are laying it down as a prima facie fundamental fact about delivery relations. (What reason do we have to

reject it? See below.) We wouldn't call $\vdash_T$ a delivery relation for T if we could have that $A \vdash_T B$ and yet have true admissible extensions of T where A is true but B untrue.

Delivery relations, in virtue of what they do, afford characterization of various types of sentence. For example, sentences that are common to *every* closed theory (theories 'closed' under their respective delivery relations) are delivered by the base universal delivery relation (viz., logic); and these are just called 'logical truths'. And another sort of sentence is highlighted in this paper: namely, trivializing sentences. A sentence that trivializes a theory is one whose truth (according to the theory) delivers triviality, that is, delivers the trivial theory in the language of the theory. But, then, a natural adequacy condition on one important kind of trivializer predicate is simply the narrow-adequacy condition: for any A, the truth of A extensionally conjoined with the truth that A is a trivializer delivers triviality (according to the given delivery relation).

Contrary to the objection, the narrow adequacy condition on trivializer predicates is natural and highly plausible for at least one common usage of 'trivializing sentence'. The question, then, is not whether the existence of narrowly adequate trivializer predicates can itself be *proved* without somewhere somehow invoking a contracting delivery relation. The question – *and the chief challenge of this paper* – is to give a simple and plausible account of why the natural and highly plausible narrow-adequacy condition should be rejected.[7]

Why reject narrowly adequate trivializer predicates? The answer, in the running objection, adverts to full-on contraction freedom. But until we have a plausible philosophical story that motivates rejecting the very existence of a narrowly adequate trivializer predicate in English, full-on contraction freedom is dubious (at least if indeed required for narrowly adequate trivializer predicates). As far as I can see, there is no such story, though this paper serves as an invitation to defenders of the target semantic-completeness argument to provide one. As things stand, the only reason for rejecting the existence of a narrowly adequate trivializer predicate is that otherwise we'd be led to the conclusion that the true (English) story of English is trivial – *or incomplete*. But such a 'reason' to reject narrowly adequate trivializer predicates (viz., to avoid the second limitative disjunction) is at best question-begging in the context.

Parenthetical remark. I note that Priest's recent philosophical remarks about 'fusion and confusion' [11] do not, as far as I can see, clearly *motivate* a rejection of narrowly adequate triviality predicates. Moreover, while delivery relations (or closure relations) for theories in general are entailment relations, it appears that Priest's philosophical remarks (and similarly those of Weber [19]) chiefly concern *logical entailment* – not the critical issue of closure on would-be complete semantical theories in general. Moreover, and importantly, the semantic story that Priest gives [11], which breaks contraction via impossible points at which logically impossible behavior occurs, will not apply to theoretical delivery re-

[7] A related challenge is to spell out exactly what theories and their closures are supposed to be if they're not something that reflects narrow adequacy of the delivery relation. Some answers to this challenge face very clear problems, as discussed by Ripley [12].

lations on pain of failing to deliver even elementary logical consequences into the theory. Such semantical accounts of breaking contraction may be plausible for a variety of important relations; but they're too liberal, with respect to what counts as a candidate counterexample, to characterize a theory's overall delivery relation. Once a delivery relation counts impossible points among its relevant counterexamples, the delivery relation will be implausibly weak. (For example, there's no non-ad-hoc reason not to in turn allow 'counterexamples' in which impossible conjunction or disjunction behavior occurs. But once one allows such points to count as candidate counterexamples for a theory's delivery relation $\vdash_T$, one then breaks, for example, the delivery of B from $A \wedge B$. But Priest certainly accepts that our true theories – and certainly the would-be semantically complete theory of English – has a delivery relation that delivers B from $A \wedge B$. So, the story of impossible points doesn't apply to target delivery relations, and hence doesn't explain why we should reject narrowly adequate trivializer predicates.)

Finally, I note that perhaps the philosophical story advanced by Lionel Shapiro [16] can be filled out in a way that motivates the rejection of narrowly adequate triviality predicates; but that story, even in outline, appears to be very different from the semantics-heavy views reflected in the target semantic-closure arguments. Moreover, Shapiro's framework – much like that of Zardini [21] – purports to afford avenues for avoiding glut theory, and so does not help the defenders of the target glut arguments in the present dialectic. *End remark.*

8 Concluding remarks: prospects for paradox-driven glut theory

I have argued against the viability of certain well-known and historically important arguments for glut theory. I have argued against the route towards glut theory via familiar limitative-disjunction results.

One might wonder whether glut theory retains any strong motivation from paradox once the promise of semantic completeness – the promise of giving the full truth about English in English – is removed.[8] In fact, I think that there *is* an important motivation for paradox-driven glut theory: namely, *naturalness*.

In short, we should accept that some (not all!) paradox-looking sentences are gluts because the most natural default treatment of them is a glutty one. The prime examples, of course, are liar sentences involving (for example) a merely transparent or see-through truth predicate (where an attribution of such truth to sentence A is everywhere-non-intensional intersubstitutable with A). Such naturalness motivates gluts even in the absence of excluded-middle-like behavior – motivates, though doesn't logically (or otherwise) force the conclusion. The glutty treatment arises from aesthetic considerations: such target liars simply look like gluts.

[8] Priest [10] and Routley/Sylvan [17] have given many other alleged sources for glut theories. I myself reject those, but each such case requires individual reply – a task for elsewhere. This paper tackles only the best-known argument (viz., from semantic closure).

Naturalness as a source for gluts requires both elaboration and defense, a task I take up elsewhere. I flag it here only to rebut the idea that there are no viable paradox-driven motivations for glut theory now that, as I have argued, the well-known argument from the promise of semantic closure to gluts fails.[9]

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