

# Truth Without Detachment

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- ▶ Aim 2: frame the discussion in terms of 'Feferman's challenge' to non-classical truth theories (focus: LP).
- ▶ *Take-home 1*: that familiar non-classical theories face a prima facie challenge is uncontroversial; the debate is over *how best* to meet the challenge.
- ▶ *Take-home 2*: there is an alternative to the 'quest for suitable conditionals', one that both distinguishes Logic from reasoning [10] and recognizes the role of extra-logical principles in acts of inference.

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 Definition: let  $A \supset B$  be  $\neg A \vee B$
  - ▶ **Detachment**:  $A, A \supset B \not\vdash_{lp} B$ .
- ▶ In general: LP is detachment-free [6]!
  - ▶ Connective  $\odot$  is detachable in logic L iff  $A, A \odot B \vdash_L B$ .
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...*Good news* is that we can coherently enjoy a 'transparent truth' predicate in an LP (similarly, K3) setting; **but...**



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*Feferman's challenge*: explain how it is that 'sustained ordinary reasoning can be carried out' if our logic is so weak.

This challenge (though not by this name) has been embraced by non-classical truth theorists for the last 50 years or more through today. The standard response: recognize additional logical machinery... Prepare for the quest!

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  - ▶ **Trouble**: Curry's paradox via  $A \wedge (A \rightarrow B) \rightarrow B$  etc!

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Standard PMP derivation of Curry's paradox [15]:

1.  $C \leftrightarrow (C \rightarrow \perp)$ . [T-biconditionals]
2.  $C \wedge (C \rightarrow \perp) \rightarrow \perp$ . [**PMP**]
3.  $C \wedge C \rightarrow \perp$ . [2; Substitution Equivalents]
4.  $C \rightarrow \perp$ . [3; Conjunction behavior]
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*...back to Quest – giving up PMP (etc)!*

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  - ▶ **Trouble**: Curry's paradox via  $A \wedge (A \rightarrow B) \rightarrow B$  etc!
- ▶ Standard route: go 'non-extensional'! Generally: use a strict-implication approach in a non-normal-worlds or 'impossible-worlds' setting. [Details aside]

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...but if we give up the quest, where do we go?

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Rejecting inconsistency: we also – all of us – accept principles whereby we reject gluts (indeed, without thinking about it). At the very least, it is an exceptional case where we accept a glut.

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*...but there are some notable  $LP^+$  validities corresponding to the LP invalidities...*

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Consider the noted **LP invalidities**: DS and Detachment.

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[Dual K3:  $X \vdash_c^+ Y$  iff  $e(Y) \cup X \vdash_{k3}^+ Y$ , where  $e(Y)$  is the 'atomic completeness set of  $Y$ ', containing  $p \vee \neg p$  etc.]

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Question: how do we choose among the options that Logic gives us? ***Answer: we resort to extra-logical principles!***

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[Dual K3: we accept the pivotal 'completeness' claims in  $e(Y)$ .]

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- ▶ ...but  $\{A, (A \supset B)\}$  does imply  $\{B, A \wedge \neg A\}$ .

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- ▶  $\{A, (A \supset B)\}$  does not imply  $B$ . Period.
- ▶ ...but  $\{A, (A \supset B)\}$  does imply  $\{B, A \wedge \neg A\}$ .
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Logic has left us with a 'choice', and *extra-logical* principles against accepting contradictions go from there.

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[nb: this is dual to the paracomplete case, where extra-logical principles tell us to *accept* LEM instances.]

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  - ▶ Liar is *rare* exception: go back and choose the glut!

# DISCUSSION TIME...

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*...Thank you for your time!*





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