Time for curry

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This paper presents a new puzzle for certain positions in the theory of truth. The relevant positions can be stated in a language including a truth predicate T and an operation $\ulcorner\ \urcorner$ that takes sentences to names of those sentences; they are positions that take the T-schema $A \leftrightarrow T(\ulcorner A \urcorner)$ to hold without restriction, for every sentence A in the language. As such, they must be based on a nonclassical logic, since paradoxes that cannot be handled classically will arise. The best-known of these paradoxes is probably the liar paradox – a sentence that says of itself (only) that it is not true – but our concern here is not with the liar. Instead, our focus is a variant of Curry's paradox [3, 5, 7, 11] – a sentence that says of itself (only) that if it is true, everything is true.

§1 is necessary stage setting: we present the standard version of Curry's paradox and the strain of response to it we wish to focus on. This strain of response crucially invokes *non-normal worlds*: worlds at which the laws of logic differ from the laws that actually hold. In §2, we go on to argue that, in light of *temporal curry paradox* (a novel version of curry paradox that we present here), this strain of response ought also to accept *non-normal times*: times at which the laws of logic *in the actual world* differ from the laws that hold now. We then consider, in §3, what this would mean for the theorists in question.

1 The state of play

1.1 Curry paradox

By the usual diagonalization methods, there is a sentence C that is equivalent to $T(\ulcorner C \urcorner) \to \bot$, where $\to$ is the (detachable) conditional in the T-schema.[1] Here, take $\bot$ to be an 'explosive sentence' from which everything follows. (It might be 'everything is true', or some such.) Given these resources, a proof from the T-schema to $\bot$ threatens (where $\leftrightarrow$ is defined via $\wedge$ and $\to$ as usual):

1.	$T(\ulcorner C \urcorner) \leftrightarrow C$	T-schema
2.	$T(\ulcorner C \urcorner) \leftrightarrow (T(\ulcorner C \urcorner) \to \bot)$	1, substitution
3.	$T(\ulcorner C \urcorner) \to (T(\ulcorner C \urcorner) \to \bot)$	2, $\wedge$-elim
4.	$T(\ulcorner C \urcorner) \to \bot$	3, contraction
5.	C	4, substitution
6.	$T(\ulcorner C \urcorner)$	1, 5, modus ponens
7.	$\bot$	4, 6, modus ponens

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[1]The conditional involved in the standard Curry sentence is the same conditional used in the T-schema, and theorists differ on just how this conditional ought to behave. For our purposes here, we focus only on *detachable* conditionals – conditionals that validate modus ponens. (By 'modus ponens' we mean – throughout – only the so-called rule form: that A and $A \to B$ jointly imply B, that is, that the *argument* from $\{A, A \to B\}$ to B is valid.)

Here, the step from 3 to 4 is justified by contraction: from $A \to (A \to B)$ we can conclude $A \to B$.[2] Since substituting equivalents, eliminating $\wedge$s, and modusing ponens all seem like surer steps than contraction, one plausible way to treat this paradox while retaining the T-schema is to provide a theory of $\to$ on which contraction is not a valid inference. Such theories have recently been advanced by Beall [1], Brady [2], Field [4], Priest [10], Sylvan [15], and others.

1.2 Explanation and truth conditions for conditionals

Among these theorists, some think that contraction's invalidity is to be explained by $\to$'s truth conditions, typically because validity and invalidity in general are taken to be matters of truth conditions. For example, Priest argues that 'validity is the relationship of truth-preservation-in-all-situations' [9, ch. 11], and this thought is also forcefully embraced in the work of Routley [14, Appendix I], and also evident in Brady's work [2]. While not universally endorsed,[3] the thought is natural and common in philosophy. Example: why is it that (say) $A \wedge B$ implies B? Answer: the definition of implication (validity) is 'truth preservation over all conditions', and the conditions in which a conjunction is true (i.e., the truth conditions for a conjunction) have it that $A \wedge B$ is true just if both A and B are true. The explanation falls out of truth conditions and their role in validity. Likewise, the explanation for the *failure* of $A \vee B$'s implying B invokes the truth conditions for $\vee$ and the existence of situations in which $A \vee B$ is true and B not. And the same goes, according to target theorists, for contraction.

Among such theorists, the dominant approach to truth conditions for $\to$ invokes frames involving points (worlds or world-like entities). We consider such an approach here; and we call the points 'worlds' without worrying what they are. As a first approximation, let a frame be a set W of worlds, and let a model be a frame together with a relation $\Vdash$ between worlds and sentences of our language. $\Vdash$ can hold or not between any world and any (non-logical) atomic sentence, but it is constrained for compound sentences. On the approach we are considering, it is these constraints that give sentential connectives their meanings. For example, here are constraints to give $\wedge$, T, and $\perp$ their meanings:

$$w \Vdash A \wedge B \text{ iff } w \Vdash A \text{ and } w \Vdash B$$

$$w \Vdash T(\ulcorner A \urcorner) \text{ iff } w \Vdash A$$

$$w \nVdash \perp, \text{ for any } w$$

It is sometimes useful to distinguish *extensional* from *intensional* connectives. Extensional connectives don't look across worlds; whether a sentence built from an extensional connective is satisfied at a world depends only on what else is satisfied at that world. Intensional connectives, on the other hand, look across worlds; whether a sentence built from an intensional connective is satisfied at a world can depend on what happens at other worlds. $\wedge$, T, and $\perp$ are all

[2] This use of 'contraction' is related to, but importantly distinct from, its use to describe the structural rule that allows for repeated use of premises. For example, in the above proof, premise 1 is used twice (in the justifications of steps 2 and 6); this involves appeal to the structural rule, but not to the $\to$-related rule we call 'contraction', which is involved above only in the step from 3 to 4.

[3] For example, Beall [1] and Field [4] reject the explanatory role of truth conditions.

extensional. Since $\rightarrow$ will be used to express the strong T-schema connection between A and $T(\ulcorner A \urcorner)$, its constraint will take into account the relation between its antecedent and consequent across worlds, and so $\rightarrow$ is intensional:

$$w \Vdash A \rightarrow B \text{ iff for all } w' \in W, \text{ either } w' \nVdash A, \text{ or } w' \Vdash B.$$

Now we can define validity. An argument from a set of sentences Γ to a set of sentences Δ is valid ($\Gamma \vDash \Delta$) iff in every model on every frame, at every world w such that $w \Vdash A$ for every $A \in \Gamma$, $w \Vdash B$ for some $B \in \Delta$. (This is the general, multiple-conclusion relation. Restricting to singleton conclusions reduces to the usual single-conclusion account. The general account is worth having, though nothing we say here hangs on the generality.)

1.3 Contraction freedom and non-normal worlds

But there is a problem. As things currently stand, $A \rightarrow (A \rightarrow B) \vDash A \rightarrow B$. That is, contraction holds. The other principles used in the problematic Curry argument also hold. So such truth conditions can't be the whole story; they would force us to conclude $\bot$, and thus every sentence – naked absurdity.

If it is to be contraction-free, $\rightarrow$ must derive its meaning from some constraint that doesn't force contraction on it. For ideas as to how this is to be done, we can look to frames developed for weak relevant and linear logics, in which contraction fails. Here, the usual way cuts a distinction in W, namely, *normal worlds* and *non-normal worlds*. Thus, we require our frames to be slightly more articulated, specifying a set W of worlds, and a set $N \subseteq W$ of normal worlds. For any normal world $w \in N$, we constrain $\Vdash$ as before. But for any non-normal world $w \in W \setminus N$, we treat $\rightarrow$-sentences differently; for our purposes here, we allow $\rightarrow$-sentences to be satisfied or not by non-normal worlds *arbitrarily*.[4]

If this is the only shift we make, however, we lose such important validities as modus ponens. (There will be non-normal worlds at which A holds, $A \rightarrow B$ holds, and B does not hold, since whether $A \rightarrow B$ holds there has nothing to do with where or whether A or B holds anywhere.) So we must also change our understanding of validity: the argument from a set of sentences Γ to a set of sentences Δ is valid ($\Gamma \vDash \Delta$) iff in every model on every frame, at every *normal* world w such that $w \Vdash A$ for every $A \in \Gamma$, $w \Vdash B$ for some $B \in \Delta$. The restriction to *normal* w in this definition ensures that, in evaluating validity, we only look at worlds in which the $\rightarrow$ is 'well-behaved'. In particular, modus ponens is valid, given this new understanding of validity.

Crucially, however, contraction remains invalid. Although we only look at normal worlds in checking validity, those normal worlds themselves look at *all* worlds—normal and non-normal—in the truth-conditions for $\rightarrow$-sentences. As such, it's possible for $A \rightarrow (A \rightarrow B)$ to hold at a normal world without $A \rightarrow B$ holding there: this can happen if there is a (non-normal) world at which A and $A \rightarrow B$ both hold, but where B does not hold. Counterexamples to contraction *somewhere* thus rely on counterexamples to modus ponens *somewhere else*; the distinction between normal and non-normal worlds allows us to keep these

[4]There are other options; the important thing for our purposes is the distinction between normal and non-normal worlds, and that this distinction matters for the constraints on $\Vdash$ when it comes to $\rightarrow$-sentences. The details of the constraints (if any) that operate at non-normal worlds are beside the point.

somewheres organized, so that the counterexamples to contraction are sufficient to undermine its validity, while the counterexamples to modus ponens are not.

Of course, if the invocation of non-normal worlds is meant to *explain* the failure of contraction, it is not enough simply to offer this kind of model theory. The explanation must tell us something about what non-normal worlds *are*, and why they are related to $\to$ in the way the model theory takes them to be. Indeed, such a theory is offered by Priest [8, p. 15]:

> The normal worlds are to be thought of as (logically) possible worlds. Non-normal worlds are to be thought of as (logically) impossible worlds. The idea that there can be physically impossible worlds, that is, worlds where the laws of physics are different, is a standard one. Such worlds are still logically possible. But just as some worlds have laws of physics different from the actual physical laws, so some worlds have laws of logic different from the actual logical laws.

Our approach here shall assume that this—allowing for failures of contraction, to be explained by invoking worlds at which logical laws differ—is broadly the right way to address Curry's paradox, and to explore how this approach adapts to a novel version of curry paradox with slightly different ingredients.

2 The meat

2.1 Robust contraction-freedom

First, note that it is not enough just to avoid contraction for $\to$. Curry trouble arises if there is any connective $\Rightarrow$ meeting the following three conditions:

$\to$-consequence: From $A \to B$, we can infer $A \Rightarrow B$.

$\Rightarrow$-modus ponens: From A and $A \Rightarrow B$, we can infer B.

$\Rightarrow$-contraction: From $A \Rightarrow (A \Rightarrow B)$, we can infer $A \Rightarrow B$.

(For details, see Restall's discussion [13].) In fact, we can replace the first condition, $\to$-consequence, with the condition:

$\Rightarrow$-T-schema: $A \Leftrightarrow T(\ulcorner A \urcorner)$ is provable.

From $\to$-consequence and the ($\to$-involving) T-schema, $\Rightarrow$-T-schema follows. And the Curry proof in §1.1 for $\to$ can simply be repeated as is for $\Rightarrow$ if $\Rightarrow$-T-schema, $\Rightarrow$-modus ponens, and $\Rightarrow$-contraction all hold.

Below, we present a connective that, at least prima facie, seems to satisfy $\Rightarrow$-T-schema, $\Rightarrow$-modus ponens, and $\Rightarrow$-contraction. We then argue that a friend of non-normal worlds explanations of the sort mentioned in §1.2 ought to acknowledge non-normal *times* to address this threat.

2.2 Temporal Curry

Sentences are not just true at some worlds and false at others; they can also be true at some times and false at others, even within a single world. To accommodate this, we follow Kaplan [6] and expand our models. We now take our models to specify a set W of worlds and a set T of times. As before, we divide W into the normal worlds N and the others, the non-normal worlds. Now, we can specify truth conditions for our connectives almost as before. The difference is straightforward: sentences are not true-at-worlds, but instead true-at-world/time-pairs. For all of our above connectives, the truth conditions change only slightly: they now carry an idle time parameter. For example: for any $w \in W$ and any $t \in T$:

$$\langle w, t\rangle \Vdash A \wedge B \text{ iff } \langle w, t\rangle \Vdash A \text{ and } \langle w, t\rangle \Vdash B.$$

The conditional $\rightarrow$ is perhaps of more interest, but the same idea applies. For *normal* worlds w:

$$\langle w, t\rangle \Vdash A \rightarrow B \text{ iff for all } w' \in W \text{, either } \langle w', t\rangle \nVdash A, \text{ or } \langle w', t\rangle \Vdash B.$$

As before, at non-normal worlds $\Vdash$ is not constrained for $\rightarrow$-sentences; here, this lack of constraint extends to all times.

If this were all there were to temporal models, they would not be very interesting. They come into their own when we consider connectives that shift the time parameter. By analogy with the 'extensional'/'intensional' terminology to describe whether a connective shifts the world parameter, we can draw a distinction between 'extemporal' and 'intemporal' connectives. All of our old connectives are extemporal, but intemporal connectives allow us to use the structure that temporal models provide. The most familiar intemporal connectives are unary connectives, studied by Prior [12], often written F, G, P, and H. Here, we skip these, to explore the behavior of a binary intemporal connective, which we write (only for lack of obviously better notation) as a short map arrow: $\mapsto$. Informally, $A \mapsto B$ is to be read as something like 'whenever A, B'. This informal reading is evident in the truth-conditions:

$$\langle w, t\rangle \Vdash A \mapsto B \text{ iff for all } t' \in T \text{, if } \langle w, t'\rangle \Vdash A, \text{ then } \langle w, t'\rangle \Vdash B.$$

Despite being intemporal, $\mapsto$ is extensional: its range of truth values at a world w does not depend on any world beyond w.

But now trouble is brewing. Call a world-time pair $\langle w, t\rangle$ a *normal-world pair* just if $w \in N$. Presumably, we want it to be the case that $A \mapsto T(\ulcorner A \urcorner)$ and $T(\ulcorner A \urcorner) \mapsto A$ are logical truths: true at all normal-world pairs. Indeed, as far as we can see, any motivation for the T-schema's validity at *worlds* is equal motivation for its validity at *times*. (Just as it's very difficult to imagine a world at which A holds without $T(\ulcorner A \urcorner)$ holding and vice versa, so too it's very difficult for *times*.) But now the trouble bubbles to the surface. In particular, notice that, given the truth conditions for $\mapsto$, we have both $\mapsto$-*modus ponens* and $\mapsto$-*contraction*. Hence, mixed with $\mapsto$-*T-schema*, we have the ingredients for explosive curry (see §2.1). This is a *temporal* curry paradox that's as explosive as its standard non-temporal relative.

2.3 The bite: non-normal *times*

Since this is substantially the same problem as Curry's paradox for $\rightarrow$, we think it should receive substantially the same solution. In short, $\mapsto$ obeys the given truth conditions (see §2.2) at *most* world-time pairs; however, there are world-time pairs – call them *abnormal* – at which $\mapsto$ fails to conform to the given truth conditions. (Perhaps, as with $\rightarrow$, the behavior of $\mapsto$ is arbitrary at abnormal pairs.) Such abnormal pairs involve 'non-normal times', *times* at which laws of logic fail.

The bite is more than that there be *some* world-time pair $\langle w, t\rangle$ that is abnormal in having a 'non-normal time'. The bite is stronger: *every* world – and, hence, every *normal* world, including this (our actual) one – features in some abnormal pair. Suppose otherwise: fix a world w and suppose that for all $t \in T$, the 'whenever' connective $\mapsto$ obeys the given truth condition at $\langle w, t\rangle$. Then we have curry trouble at w. Consider a curry sentence C equivalent to $T(\ulcorner C \urcorner) \mapsto \bot$. Suppose $\langle w, t\rangle \Vdash C$. Then for all $t' \in T$, if $\langle w, t'\rangle \Vdash T(\ulcorner C \urcorner)$ then $\langle w, t'\rangle \Vdash \bot$. Since $\langle w, t'\rangle \nVdash \bot$ for all t', it must be that $\langle w, t'\rangle \nVdash T(\ulcorner C \urcorner)$ for all t'. But then we have a counterexample at $\langle w, t\rangle$ to the $\mapsto$-T-schema. This cannot be. Hence, for all t, we have $\langle w, t\rangle \nVdash C$. If we are to avoid a counterexample to the $\mapsto$-T-schema, it must be that for all t, $\langle w, t\rangle \nVdash T(\ulcorner C \urcorner)$. But, then, by $\mapsto$'s truth conditions, $\langle w, t\rangle \Vdash T(\ulcorner C \urcorner) \mapsto \bot$ for any t. So this is impossible too.

The philosophical rub comes out when considering the actual world. As above, for any w there must be abnormal pairs $\langle w, t\rangle$ at which $\mapsto$ does not obey the given truth conditions. Consider the actual world @, and let the *non-normal times* be those times t for which $\langle @, t\rangle$ is an abnormal pair. By the argument above, there must be non-normal times. But this is philosophically awkward: it is much harder to make satisfying philosophical sense of non-normal times than it is of non-normal worlds.

Non-normal worlds, recall, are worlds where the actual laws of logic do not hold. Since worlds are unfamiliar and odd sorts of places anyhow, it is not so challenging to suppose that some of them fail laws of logic in this way. But if abnormal pairs are pairs where laws of logic do not hold – as they must be – then there must be times at which laws of logic fail *in the actual world*. This, we think, is harder to swallow. There is no modal cushion between us and the failure; it is only a matter of minutes. (It may be many minutes; maybe all of the failures are tucked away safely in the past, or far off in the future. But still, they must be there—here!—even if not now.) This failure is serious: as we saw before, for contraction to fail *here*, modus ponens must fail *somewhere*. Thus, there are times at which modus ponens fails in the actual world.

3 Possible responses

Of course, one can simply bite the bullet and admit that there are non-normal times. Perhaps this is a discovery rather than a reductio. (Certainly, when paradoxes are in the air, one has been mistaken for the other before.) But if this is the right way to understand the situation, more needs to be said to assuage the initial awkwardness. Even those of us who were prepared to go along with non-normal worlds feel some difficulty allowing for non-normal times. A story about why logical laws might change over time, analogous to the way they can be taken to change over worlds, would be a great help to resolve this difficulty.

Another possibility would be to attack the analogy we have exploited between worlds and times. For example, perhaps there is some reason why there should be no extensional intemporal connectives like $\mapsto$, despite the presence of intensional extemporal connectives like $\rightarrow$. We don't immediately know what such a reason could be. However, if there were some reason that anything like

$\mapsto$ had to be intensional, we could invalidate contraction at this world by invalidating modus ponens at some other (presumably non-normal) world. Then there would be no need to invoke non-normal times to avoid temporal curry. Again, though, more would need to be said to make this plausible.

To sum up: if the failure of $\to$-contraction is to be explained by $\to$'s relying on worlds at which logical laws fail, then the failure of $\mapsto$-contraction ought to be explained by $\mapsto$'s relying on times at which logical laws fail. At least prima facie, however, allowing for actual times at which logical laws fail is quite awkward, more awkward than allowing for non-actual worlds at which logical laws fail. So the advocate of non-normal worlds must either 1) explain why they do not advocate non-normal times, or 2) explain why non-normal times are not as awkward as they first appear. We see no obvious way to do either of these, and so we leave this dilemma, at least for now, as a dilemma.

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